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Measuring aspects of quantitative judgment of upper elementary and junior high school students.

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MEASURING ASPECTS OF QUANTITATIVE JUDGMENT OF UPPER ELEMENTARY AND JUNIOR HIGH SCHOOL STUDENTS

A Dissertation Presented

By

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CHAPTER I

CHARACTERISTICS OF THE STUDY

Introduction

It has become clear than all of the classical branches of mathematics have profited from the recent advances exhibited in analysis, topology, and abstract algebra. As an example, Cantor recognized in the late nineteenth century that an adequate theory of sets must be developed in order to clarify and make possible the solutions of numerous problems in analysis and algebra. Thus, the basic nature of the set concept is readily acknowledged as an integral part of many elementary mathematics programs.

In the realm of current and past developments, it becomes readily apparent that a mathematical system assimilates an abstract form which may occur frequently as the underlying pattern in many settings. Hence, the mathematician and the teacher of mathematics must condition the student so that one becomes less concerned with the solution of specific problems than with the development of general patterns that have widespread applicability in the study of particular situations.

A major problem confronting mathematics educators is the development of techniques which will help one to identify factors and concepts which tend to hinder a student's learning ability.

Hence, one of the most important aspects of mathematics teaching is the role of evaluation in the improvement of instruction. Through evaluation one will be able to determine the effectiveness of the techniques and materials used in teaching.

Traditionally, algebra one and geometry have been the mathematics courses in which the student has been confronted with the axiomatic structure of mathematics for the first time. Some students perform exceptionally well in this situation while others experience varying degrees of difficulty.

Some of the goals of the Cambridge Conference on School Mathematics were concerned with the presentation of the axiomatic structure of mathematics to a greater percentage of students at a younger age. An interesting contribution to education, for the evaluation of students, would be the development of an instrument which would measure the quantitative factors intrinsic to judgment and intuitive powers possessed by the student. Hopefully, this instrument can be used as an effective measurement of quantitative judgment which may assist in placing students in appropriate classroom situations commensurate with their understanding, attitudes, problem solving ability, and logical reasoning capacity.

Justification of the Study

Relatively little research has been conducted concerning the student's ability to use quantitative thinking or judgment efficiently or effectively. Early studies by Sueltz[1], Martin[2], and Wilson[3] have produced some insights into students' abilities to deal with quantitative judgments with respect to social situations. Martin stated that children become more adept at handling quantitative concepts with increases in age. Yet, more needs to be known about the significance of the relationship between intelligence, achievement test scores, previous mathematics grades, and grade level.

Hall[4], using his self-developed Test of Quantitative Judgment, made the following conclusions based on his sample of over 700 children in grades four, five, and six. First of all, he found no significant ability differences with respect to sex when dealing with aspects of quantitative judgment. Secondly, he reported that children's ability to deal with aspects of quantitative judgment varied directly with increasing grade level. In addition, when the level of

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[1]Ben A. Sueltz, "The Measurement of Understanding and Judgments in Elementary School Mathematics," The Mathematics Teacher, Vol. XL (October, 1947), pp. 279-284.

[2]William E. Martin, "Quantitative Expression in Young Children," Genetic Psychology Monographs, Vol. XLIV (November, 1951), p. 214.

[3]Guy M. Wilson and Mabel Cassell, "A Research in Weights and Measures," Journal of Educational Research, No. 46 (April, 1953), pp. 575-585.

[4]Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment," (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), pp. 87-88.

intellignece was held constant, the correlations between quantitative judgment and arithmetical ability by sex and grade are low. This indicates that quantitative judgment is a unique attribute. Finally, the estimated reliabilities indicate that there is crudeness in the instrument developed to measure quantitative judgment. Yet, Hall felt that it was considerably better than the usual teacher-made tests and it undoubtedly does measure the ability of children on aspects of quantitative judgment.

Tuttle[5] conducted a pilot study concerned with the refinement and the improvement of the Test of Quantitative Judgment developed by Hall.

Sister Josephina[6] related that research studies, though limited, indicate that young children show an early interest in number concepts. She conducted a pilot study to determine the level of the child's arithmetical knowledge by randomly selecting thirty children, ages four and five, who were enrolled in a modified Montessori program. As a consequence of the scores on the Stanford-Binet Test of Intelligence (Form L-M) and her self-constructed test of arithmetic, Sister

[5]Cynthia L. Tuttle, "The Refinement of a Test of Quantitative Judgment," (unpublished M.Ed. thesis, School of Education, University of Massachusetts, 1965).

[6]Sister Josephina C.S.J., "Quantitative Thinking of Pre-School Children," The Arithmetic Teacher, Vol. XII (January, 1965), pp. 54-55.

Josephina asserted that pre-school children possess quantitative ability to a degree which needs the attention of curriculum specialists and teachers. She recognized that the sample was limited and that the intelligence quotient was above average (137 with a sigma spread of 11.5). Furthermore, she reported that the results of her arithmetical knowledge test demonstrated that much arithmetical knowledge is learned incidently, since these children had not been formally taught number concepts and quantities.

Wick[7] found that the high school mathematics record was consistently the best predictor of success in first year college mathematics.

Friebel[8] conducted an investigation of the mastery and understanding of measurement among students enrolled in "modern school mathematics". Data included in the report seemed to justify the following relevant conclusions: (1) Newer programs tend to promote significantly superior growth in arithmetic reasonings; (2) Modern school mathematics may be more effective in promoting learning associated with measurement; (3) Modern mathematics students had significantly greater ability in the process of estimation of measures dealing with area and volume.

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[7]Marshall E. Wick, "A Study of the Factors Associated with Success in First Year College Mathematics," The Mathematics Teacher, Vol. LVIII (November, 1965), pp. 642-648.

[8]Allen C. Friebel, "Measurement Understandings in Modern School Mathematics," The Arithmetic Teacher, Vol. XIV (October, 1967), pp. 476-480.

Madden[9] hypothesized that there was a need to establish for each student his most "fertile zone of instruction". A highly promising new direction for the purpose of defining a student's "fertile zone of instruction" would be the combination of tests of learning potential in mathematics, not general potential, with achievement tests in mathematics. This zone would span the curriculum commencing where the student can learn mathematics on his own initiative to an upper limit where he needs professional instruction to assist his comprehension.

Thus evaluation of mathematical competence needs to include measures of fluency in applications and in number facility. Of greater importance is the measurement of the tendency and ability to think as mathematical people think, to define the problems precisely, to state them succinctly, and to relate them quantitatively to whatever is appropriate.

The primary challenge for new directions at the elementary and junior high levels lies in analyzing the deeper mental processes of children and in mapping the possibilities and judicious limits for stimulating maturation into these deeper, mathematical, mental structures.

Definition of Terms

In this study it will be appropriate to define the

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[9]Richard Madden, "New Directions in the Measurement of Mathematical Ability," The Arithmetic Teacher, Vol. XIII (May, 1966), pp. 375-379.

following two terms: (1) "Quantitative thinking or quantitative judgment," which has been defined by Hall[10], "will refer to the individual's ability to apply number, mathematical concepts, and mathematical processes to quantitative situations encountered socially both within and outside the classroom environment; (Quantitative judgment includes thinking about amounts, estimating or guessing intuitively relative to how much, how many, how far and/or how large.)" (2) Normal social environment is to mean the usual situations where number, or the concept of number is found.

Statement of Problem

The problem includes the following: (1) To measure the ability of upper elementary and junior high school students on aspects of quantitative judgment relative to their normal social environments; (2) To determine the relationships between students' abilities in dealing with quantitative judgments in relation to their sex, Iowa Achievement Scores, intelligence quotients, grade levels, and their mathematics grades for the previous two years; (3) To determine if there exists an important amount of variance remaining after removing the variation contributed by the other measures in the study (I.Q. test, Iowa tests, grade level, and previous mathematics

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[10]Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment," pp. 4-5.

grades).

Scope of the Study

The study was conducted in four elementary schools and a junior high school in a Western Massachusetts city. Ten sixth grade classes, eight seventh grade classes, nine eighth grade classes, and eleven ninth grade classes were utilized in the study. There was an overall total of 884 cases in the study; a population of 249 students in sixth grade, 210 students in seventh grade, 208 students in eighth grade, and 217 students in ninth grade. The study was restricted to those classrooms composed primarily of average and above-average students.

TABLE 1

SUMMARY BY GRADE AND SEX OF POPULATION USED IN THE STUDY

Grade	Girls	Boys	Total
6	117	132	249
7	109	101	210
8	104	104	208
9	120	97	217
Total	450	434	884

Table 1 indicates the population subgroups of the study.

CHAPTER II

REVIEW OF RELATED RESEARCH

Relatively little research has been reported about the subject of quantitative judgment. Hence, much of this chapter will pertain to the research in related topics.

The growth of knowledge is not a linear relation but an exponential relation (ie., a relation involving the variable as an exponent). While the man on the street is not consciously aware of this accelerated change, particularly in the field of mathematics, it certainly should be apparent to most of us. Forcing awareness to this situation is a major portion of our problem.

Over half of the mathematical topics currently being studied are the product of twentieth century mathematicians. The primary tool for obtaining knowledge and arriving at conclusions in mathematics is the deductive method; although much of mathematics is discovered or invented inductively, mathematics is still the science of deductive reasoning. It is apparent that one needs to understand mathematical methods and the language of mathematics in order to apply them to the physical, biological, and social sciences.

Fehr[1] relates that mathematics has always held a primary position in the curriculum of the schools, chiefly because

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[1]Howard F. Fehr, "Modern Mathematics and Good Pedagogy," The Arithmetic Teacher, Vol. X (November, 1963), pp. 402-403.

it has been a necessary component in the formation of a liberally educated person. Through the ages mathematics has played an important role in precisely this sense, and today it has greater potential than ever for contributing to liberal education. But if the educated person, the desirable end product of our elaborate educational process, is to understand mathematics, one must not be left a few hundred years behind in its content and conceptualization. We are indeed under obligation to eliminate the outmoded and unimportant parts of mathematics, no matter how hallowed by tradition. Such steps are deemed necessary if we expect our students to pursue mathematics with interest and enthusiasm; outmoded concepts should be replaced by more recent, general, and powerful concepts.

Discovery Learning and Related Factors

Ausubel[2] related that in mathematics, as in other scholarly disciplines, students acquire subject matter knowledge largely through meaningful receptive learning of presented concepts, principles, and factual information. Furthermore, Ausubel states:

> The distinction between reception and discovery learning is not difficult to understand. In reception learning the principle content of what is to be learned is presented to the learner in more or less final form. The learning does not involve any discovery on his part. One is required

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[2]David P. Ausubel, "Faciliting Meaningful Verbal Learning in the Classroom," The Arithmetic Teacher, Vol. XV (February, 1968), p. 126.

> only to internalize the material or incorporate it into his cognitive structure so that it is available for reproduction or other use at some future date. The essential feature of discovery learning, on the other hand, is that the principle content of what is to be learned is not given but must be discovered by the learner before one can internalize it; the distinctive and prior learning task, in other words, is to discover something. After this phase is completed, the discovered content is internalized just as in reception learning.

In addition, the existence of the capability to engage in discovery learning depends upon the possession of certain levels of intellectual maturity and subject matter sophistication. This trait cannot be assumed to be possessed by average elementary school students, older intellectually retarded students, or neophytes in an area of specialization regardless of their degree of intellectual maturity and sophistication.

Hendrix[3] agrees the aforementioned problem concerning discovery learning arises from the failure to distinguish discovery from communication. She relates that the student's conceptualization of the appropriate answer depends, for the most part, upon the student's ability to distinguish between discovery and communication.

Madden[4] injects that an interesting concomitant of the newer mathematics programs is the promotion of the discovery method as the best way to teach it. The evaluator of

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[3]Gertrude Hendrix, "Learning by Discovery," The Mathematics Teacher, Vol. LIV (May, 1961), pp. 290-299.

[4]Richard Madden, "New Directions in the Measurement of Mathematical Ability," The Arithmetic Teacher, Vol. XIII (May, 1966), pp. 375-379.

mathematical ability must ascertain the reasons for using the discovery method. In teaching of science one supposes that discovery is an end of instruction that is related to the development of the scientific method. In mathematics it appears to be used more as a guarantee of effective retrieval of what has once been learned and an assurance of general internalization. Hence, one's knowledge of mathematics may be enhanced by understanding the role that discovery plays in the history of mathematics. Krutetskii[5] explains that success in mathematics is based least of all on the memorization of a great number of figures, numbers, and concrete facts. It has a generalized character. Types of problems and methods of solving them, patterns of reasoning and proof, and logical patterns are quickly memorized and firmly retained by the mathematically oriented student. In regard to the recollection of concrete facts, the memory is "neutral" in relation to mathematical activities. In other words, the mathematical memory has a pronounced selective nature. It does not retain all the "mathematical" information with which the student is presented, but primarily the information which has been "gleaned" from concrete facts. This is a very economical method of storing mathematical information.

It has been found that gifted students usually recall

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[5]V.A. Krutetskii, "Mathematical Abilities in Students," Soviet Education, Vol. VIII (March, 1966), pp. 15-27.

concrete facts and relationships equally well in upper elementary grades. But if the mathematically oriented student forgets something, it usually tends to be figures and concrete facts rather than mathematical relationships. The recollection of abstract mathematical relationships acquires even greater importance as the years go by, while the recollection of concrete facts become less important.

Hartung[6] suggests that one of the most noticeable things about so-called modern mathematics is not really mathematics at all. It is the instructional method used. Much of the success of introducing material at earlier ages is due to the fact that the experimenters have adopted some form of what is widely known as "the discovery method". It has long been known that children understand better and retain longer if they acquire knowledge by a cognitive process rather than by mere memorization or habituation. Now, this method has suddenly become popular. At the same time, Hartung[7] hopes that mathematics teachers learn to use "the discovery method" with wisdom and restraint. For the student to discover everything is obviously futile because it would take too long. The student needs to be told some things so one can get on with the

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[6]Maurice Hartung, "Next Steps in Elementary School Mathematics," Theory into Practice, Vol. III (April, 1964), pp. 66-70.

[7]Hartung, "Next Steps in Elementary School Mathematics," pp. 66-70.

job of learning other things. Consequently, curriculum workers in the next few years should focus much attention on the selection of those topics that yield optimum results when approached by the discovery method.

Moore and Cain[8] explain that one of the claims of the proponents of the new mathematics programs has been that, by placing less emphasis upon the mechanics of computation, new mathematics programs are able to improve the development of logical reasoning ability. They suggested that the improvements in the development of logical reasoning ability were a consequence of the increased use of mathematical rigor coupled with the use of discovery and other inductive processes. The results of their study indicated that within the new mathematics curriculum there exists educational experiences which may foster development of logical reasoning and creative thinking abilities.

Furthermore, Meder[9] reports that the activities of the current experimental programs in mathematics have been diverse and have had differing objectives. Some have sought better communication through classification of language; others have experimented with new content, or with old content placed much

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[8]William J. Moore and Ralph W. Cain, "The New Mathematics and Logical Reasoning and Creative Thinking Abilities," <u>School Science and Mathematics</u>, Vol. LXVIII (November, 1968), pp. 731-733.

[9]Albert E. Meder, "Current Experimental Programs in Mathematics," <u>Theory into Practice</u>, Vol. III (April, 1964), pp. 54-56.

earlier in the curriculum. But one startling fact recurs over and over again: Modern mathematics is not in fact, more difficult for the learner than traditional mathematics. On the contrary, being based on a more complete understanding of the nature and content of mathematics, the modern mathematics courses are likely to be more understandable and more satisfying. The basic unifying ideas of modern mathematics are easier to grasp than the complicated rules and special cases of traditional courses. To learn mathematics, it is neither necessary nor desirable to recapitulate the history of mathematical discovery; increased abstraction and rigor are not the goals--insight and power are.

Thus, one must realize the importance of having the student utilize discovery techniques. Discovery allows one to start with what one knows, which is different for each person, and build upon it. Hence, the connection between what is known and what is to be learned is the responsibility of the learner. Furthermore, one must realize that mathematical concepts are relative. Whatever one conceptualizes one perceives in its relation to other concepts derived from one's unique experience.

New concepts are more easily learned if they are conceived from previous experiences. Hall[10] mentions that

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[10]Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment," (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), p. 14.

quantitative ideas are often abstractions, and these abstractions develop into concepts only after a thorough understanding of many percepts. Once meaning and understanding have been properly fostered, the student learns to generalize from experience. It is not known whether individuals differ in amounts of innate quantitative judgment. It appears that some persons are able to judge amounts, distances, etc. more accurately than others. This may be attributed to previous training in making judgments, to environmental influences relative to amounts of concepts and generalizations, or, indeed, to some particular quality which certain persons seem to have in greater quantities than others.

Finally, Price[11] found that some tenth-grade mathematics students who were taught mathematics using specially prepared transfer materials showed a significant increase in critical-thinking ability. Furthermore, he concluded that the discovery approach by itself had no significant built-in transfer to critical thinking.

Quantitative Ideas Permeate the Study of Mathematics

Gaskell[12] stresses the existence of growing pains that lie behind the frantic outcry for more people trained in mathematics--trained beyond the levels considered adequate in the past. We have seen a revolution, not one that merely upsets tradition but a churning, heaving kind of change brought about by an avalanche of pent up demand released by advances in computation. We see now the need for a universal appreciation

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[11]Jack Price, "Discovery: Its Effect on Critical Thinking and Achievement in Mathematics," The Mathematics Teacher, Vol. LX (December, 1967), pp. 874-876.

[12]Robert E. Gaskell, "Universal Mathematical Literacy," Theory into Practice, Vol. III (April, 1964), pp. 49-53.

of mathematics and its logical and quantitative approach to problem situations. More than this, it has become evident that our standard curriculum must be compressed, pruned, and otherwise altered to make room for the more advanced material that will make it possible for our youth to contribute significantly to our advancing society.

The Cambridge Conference on School Mathematics[13] has stressed the theme of significant and relevant materials. Quantitative thinking becomes a main factor in the development of problems which are far removed from the drill of traditional texts. In addition, the conference recommended that a student who has worked through the full thirteen years of mathematics in grades K to 12 should have a level of training comparable to three years of top-level college training today; that is, we shall expect one to have the equivalent of two years of calculus, and one semester each of algebra and probability theory.

In the study of mathematics, Bidwell[14] explains that the most primitive ideas should be learned by young children in pre-school play or kindergarten. Some of these ideas include discrimination of objects, colors, manipulation of physical objects, recognition of pictures and symbols drawn on paper,

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[13] Irving Adler, "The Cambridge Conference Report: Blueprint or Fantasy?" <u>The Mathematics Teacher</u>, Vol. LIX (March, 1966), pp. 210-217.

[14] James K. Bidwell, "Learning Structures for Arithmetic," <u>The Arithmetic Teacher</u>, Vol. XVI (April, 1969), pp. 263-268.

and the initial concepts of quantitative measure. These experiences are essential for later use in developing the concepts of set, numeral, and number.

Fehr[15] insists that mathematics education, especially at the elementary school level of instruction, should not be aimed directly or solely at producing future mathematicians. These years of schooling are intended for general education--the all-encompassing intellectual development--of every schoolchild, regardless of his subsequent ambitions in life. In this general education, the uses and applications of mathematics, the needs of future scientists and humanists, the understanding by laymen, the coordination of instruction in mathematics with that of other sciences, and the need for articulating elementary, secondary, and university study are all principle factors to be considered as a new program in mathematics education is unfolded. It is now necessary to get our heads out of the clouds of recently acquired new mathematical knowledge and focus sharply on the purposes of teaching and the objectives to be gained by all children in the studies they pursue.

Research on the capacities of children of particular ages to reason logically is scarcely definitive. Inhelder and Piaget[16] have concluded that development of the capacity for

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[15] Howard E. Fehr, "Sense and Nonsense in a Modern School Mathematics Program," The Arithmetic Teacher, Vol. XIII (February, 1966), p. 85.

[16] B. Inhelder and Jean Piaget, "The Growth of Logical Thinking," translated by A. Parsons and S. Milgrin, (New York: Basic Books, Inc., 1958).

"hypothetical" reasoning or formal aspects of logic begins at about age eleven. However, a recent study by Hill[17] indicates that children of ages six, seven, and eight have a considerable grasp of many principles of logical inference, and further, they can demonstrate their understanding in reasoning from hypothetical premises. The results also indicate that simple demonstrations of correct deductions improve children's performance in the recognition of valid inference.

Hammer[18] implies that mathematics was created by people who, generally speaking, were much concerned about the durability of their work. They very much needed to know what they were talking about, and they showed a high degree of concern for the truth of their statements.

Any attempt to separate mathematics from its applications is foolish. Creative mathematical activity is not a prerogative of a few any more than creative art is. Mathematics has had amazing successes and yet remains, in its present state, principally applicable to problems of a non-complex nature. Hence, a major goal of elementary school mathematics is to assist students to see the structure of mathematics. Thus, Sandel[19] stresses the use of set concepts as the tools for

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[17] S.A. Hill, "A Study of Logical Abilities of Children," (unpublished Ph.D. dissertation, Stanford University, 1960).

[18] Preston C. Hammer, "The Role and Nature of Mathematics," The Mathematics Teacher, Vol. LVII (December, 1964), pp. 514-521.

[19] Daniel H. Sandel, "Teach So Your Goals Are Showing!" The Arithmetic Teacher, Vol. XV (April, 1968), pp. 320-323.

understanding and communicating in mathematics, and to develop control and proficiency of skills. Furthermore, the student must have an awareness of proof and the plausibility of answers if he is to develop confidence, enjoyment, and success in mathematics.

Wernick[20] explains that the mathematical part of a child's education should not be separated from the rest of his education, nor be artificially segmented within itself. The teacher must recognize the many mathematical aspects of our daily activities that are already present and available to children from their normal social environment. One must be competent and willing to introduce them into the classroom.

It should be recognized that mathematics includes much more than arithmetic. There exists a natural extension from arithmetic to algebra, from positive to negative numbers, from the number line to the coordinate plane etc. The teacher should be familiar with these extensions and present them in the appropriate way in the classroom.

The reader must realize that quantitative reasoning and quantitative judgment are not used exclusively in a mathematical oriented setting. As Young[21] has pointed out:

> The quantitative concept plays a large role in our living. Of the most common words in our language, one word in ten,

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[20]William Wernick, "An Experiment in Teaching Mathematics to Children," The Arithmetic Teacher, Vol. XI (March, 1964), pp. 150-156.

[21]William E. Young, "Teaching Quantitative Language," The Education Digest, Vol. XXII (January, 1957), p. 47.

is a mathematical term. The proportion becomes one in every four if we include indefinite quantitative words.

There exists general agreement that the most effective solutions to one's quantitative problems are those resulting from one's best reasoning and thinking. Reasoning and thinking processes of this type are naturally based upon one's understanding of mathematics. In view of this relationship Collier[22] related:

> Since man lives in a continually changing scientific world in which he is called upon more and more to make judgments and take actions relative to some quantitative aspect of daily living, it becomes imperative that we develop arithmetic understanding. If children are really to learn, enjoy, and find success in arithmetic, and in other areas of mathematics, they must understand why they do in working with numbers. They need to know the 'how' and the 'why' as well as the 'what' of arithmetic.

Relations Between Quantitative Judgment and Problem Solving

Solving problems is a complex process and growth in this process does not occur automatically as a result of daily assignments of word problems. Brown[23] stresses that slower students especially need to examine this complex operation, break it down into simpler steps and practice each step separately. Analogous to this situation, Stern and Keisler[24]

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[22]Calhoun C. Collier, "Blocks to Arithmetical Understanding," The Arithmetic Teacher, Vol. VI (November, 1959), pp. 262-268.

[23]G.W. Brown, "Improving Instruction in Problem Solving," School Science and Mathematics, Vol. LXIV (May, 1964), pp. 341-346.

[24]Carolyn Stern and Evan R. Keisler, "Acquisition of Problem Solving Strategies by Young Children, and its Relation to Mental Age," American Educational Research Journal, Vol. IV (January, 1967), pp. 1-12.

found that a strategy for solving certain types of concept identification problems can be taught to young children and there exists a postive correlation between mental age and certain problem solving strategies.

Papy explains that:

> Any child who is at present in the twelve to fifteen age group is likely to have to use mathematics later, whatever his occupation, as a means of understanding, inquiry, and problem solving.
>
> If mathematics is to be used effectively in real situations, it is not enough to have a perfect device that solves problems automatically. The first and biggest difficulty is recognizing that a situation lends itself to mathematical treatment and deciding which particular form of treatment. To do this, the concrete situation one is faced with must be conceptuatlized and mathematized. We may note in passing that most of the traditional exercises in applied mathematics do not train pupils to think in this way, which is, however, essential.
>
> Mathematics must not, therefore, be taught as though it were an isolated subject for pupils to contemplate. On the contrary, the objective from the outset is to infer it from carefully chosen situations which have a creative impact on the pupil's mind.
>
> Throughout their course of study, pupils must be persuaded to respond with an open mind to the situations given. This approach is essential whenever a real problem is to be tackled which involves using mathematical knowledge already acquired. It is essential, too, in order to assimilate any new mathematical concept. Moreover, we know that because of the rapidity with which the sciences are progressing and becoming mathematized our pupils will have at a later stage to assimilate new mathematical concepts linked to real situations. They must, therefore, be trained to keep a permanently open mind, and it is here that teaching through situations can help.

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[25]G. Papy, "Methods and Techniques of Explaining New Mathematical Concepts in the Lower Forms of Secondary Schools," Part 1, The Mathematics Teacher, Vol. LVIII (April, 1965), pp. 345-352.

Anderson[26] reported that students who are taught to understand meanings and relationships in the formation of generalizations will be more proficient at solving problems arising from quantitative situations.

Hagaman[27] injects that children learn best when they understand the meaning of what they are learning, in terms of their own experiences and interests. There are two kinds of meanings in arithmetic. One is the intrinsic meaning of the quantitative relationships which underlie mathematical thinking. The other is the functional meaning, connected with children's experiences. Both kinds of understandings are essential in arithmetic teaching. The intrinsic involves the abstract meanings of elementary mathematics, and the functional applies to practical, concrete situations.

Johnson[28] notes that a basic objective of problem solving is the discovery and generalizing from certain information by the individual.

A generalization can be thought of as a synthesis of two or more previously learned concepts. However, it is often much more than just the syntheses of these concepts, since it

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[26] G. Lester Anderson, "Quantitative Thinking as Developed Under Connectionist and Field Theories of Learning," Learning Theory in School Situations, Studies in Education, No. 2 (Minneapolis, Minn: The University of Minnesota Press, 1949), p. 69.

[27] Adaline P. Hagaman, "Word Problems in Elementary Mathematics," The Arithmetic Teacher, Vol. XI (January, 1964), pp. 10-11.

[28] David C. Johnson, "Unusual Problem Solving," The Arithmetic Teacher, Vol. XIV (April, 1967), pp. 268-271.

involves some new learning. While the "level" of generalization by individuals may vary a great deal in verbalization, symbolization, and applicability, the psychology of learning suggests that discovery of a generalization is of much more value to the learner than merely memorizing it. Emphasis on discovery and generalization makes the learning more permanent and provides for ease in transfer. Possession and recall of generalizations that suit the situation under consideration are two factors upon which a learner's success in complex problem is dependent. Cohen and Johnson[29] relate that complex problem solving considers the true problems of mathematics. (A true problem is considered to be something more than a reapplication of techniques illustrated in an earlier specific example.) A true problem in mathematics can be thought of as a novel situation for the individual who is called upon to solve it. In particular, such a novel situation is one in which the path to the goal is blocked and the individual's fixed patterns of behavior or habitual responses are not sufficient for removing the block. Hence, deliberation must take place. In this deliberation one can note many different kinds of behavior that might be exhibited by the problem solver. These behaviors can be described in such terms as the following:

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[29]Louis S. Cohen and David C. Johnson, "Some Thoughts About Problem Solving," The Arithmetic Teacher, Vol. XIV (April, 1967), pp. 261-262.

observing, exploring, decision making, organizing, recognizing, remembering, supplementing, regrouping, isolating, classifying, formulating, generalizing, verifying, and applying. Thus, as Bernstein[30] reports: "Mathematics has an inner structure and a logic which can provide a great deal of satisfaction to the individual who really understands it."

Teachers working with The School Mathematics Study Group and The University of Illinois materials have reported exciting results in terms of student interest. The use of unifying concepts to bring order and pattern into number work and logical relationships has had a very salutary effect in many teaching-learning situations.

It was proposed by Buswell[31] that, on the basis of present knowledge, it is better for teachers to make individual diagnoses of pupils' thinking in solving problems and then to help them correct fallacies in thinking and errors in number skill rather than to teach pupils to go through some set pattern of steps that is supposed to be "the way" to solve problems. Thus, the key to teaching problem solving lies in the teacher's ability to detect how pupils think as they solve their problems and then to help them to correct their errors and to think more effectively. Similarly, Brueckner and

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[30] Allen L. Bernstein, "Motivation in Mathematics," School Science and Mathematics, Vol. LXIV (December, 1964), pp. 749-754.

[31] Guy T. Buswell, "Solving Problems in Arithmetic," Education, Vol. LXXIX (January, 1959), pp. 287-290.

Grossnickle[32] said:

> The pupil must not only be able to understand the vocabulary of the statements of problems but he must also be able to visualize the situation that is presented and to sense the relationships among the quantitative elements that are involved. In addition to this he must be able to perform the necessary computations to find the answer to the problem.

Lyda and Duncan[33] conducted a study about quantitative vocabulary and problem solving. They concluded that, in terms of their sample, direct study of quantitative vocabulary contributed significantly to growth in problem solving.

Meconi[34] found that students of relatively high-ability appear to be able to learn necessary concepts for problem solving performance and retention regardless of instructional method (pure discovery, guided discovery, and rule and example). Travers[35], who devised a test of preferences for problem solving situations, related that high and low achieving students differed in their preferences of problem situations. The high-achievers had fewer preferences of any sort and favored the abstract problems more than did the low-achievers. No difference in problem-solving success between the "preferred"

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[32]Leo J. Brueckner and Foster E. Grossnickle, Making Arithmetic Meaningful, (Philadelphia, Pa.: Winston Publishing Company, 1953), p. 492.

[33]W.J. Lyda and Frances M. Duncan, "Quantitative Vocabulary and Problem Solving," The Arithmetic Teacher, Vol. XIV (April, 1967), pp. 289-291.

[34]L.J. Meconi, "Concept Learning and Retention in Mathematics," The Journal of Experimental Education, Vol. XXXVI (Fall, 1967), pp. 51-57.

[35]Kenneth J. Travers, "A Test of Pupil Preference for Problem Solving Situations in Junior High School Mathematics," The Journal of Experimental Education, Vol. XXXV (Summer, 1967), pp. 9-18.

and the "non-preferredd" problem situations was found.

Problem solving is an important process in mathematics as Grossnickle[36] connotes:

> Problem solving is taught to help the pupil discover a pattern to use in solving problems. It should not be inferred that a pupil should always follow one particular pattern in solving a problem. On the contrary, as one becomes more efficient in solving problems, one should develop many other patterns. These patterns include short cuts which indicate growth in mathematical maturity. Although it is desirable for a student to discover many different patterns for solving problems, one should have one standard form to apply to verbal problems when a ready solution is not forthcoming.

One major responsibility of mathematics instruction beginning in the elementary grades is to help pupils develop the ability to do quantitative thinking and reason logically. An understanding of mathematical theories, concepts, and relationships is vital to the solution of problems that arise in quantitative situations. If pupils are to improve in these skills, the arithmetic program should be based largely on problem solving and concepts of structure.

Vanderline[37] found that comparisons of the problem solving ability of high and low achievers implied that knowledge of vocabulary is essential to the successful solution of problems and that the study of mathematical vocabulary

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[36] Foster E. Grossnickle, "Verbal Problem Solving," The Arithmetic Teacher, Vol. XI (January, 1964), pp. 12-17.

[37] Louis F. Vanderline, "Does the Study of Quantitative Vocabulary Improve Problem-Solving?" The Elementary School Journal, Vol. LXV (December, 1964), pp. 143-152.

should be an important objective of instruction. A summary of the findings of this study are as follows:

(1) Vocabulary study should be made an integral part of the instructional program in arithmetic; (2) The study of quantitative vocabulary should begin in primary grades; (3) Pupils should be provided with more opportunities to use quantitative vocabulary in both written and oral communication; (4) Pupils should be provided with rich and varied experiences that will furnish a background for new terms to be encountered; (5) Classrooms in elementary schools should be equipped with a variety of arithmetical teaching aids to help clarify the meanings of quantitative terms.

Therefore, pupils who do not understand the technical vocabulary in each of the content areas will not comprehend many of the important concepts to be learned.

Begle[38] summarizes some of the idiosyncrasies that are usually encountered in the conceptual scheme of problem solving:

> I should like to note that any framework we may construct for mathematics education may be quite specific and not generalize to other subject matter areas. In mathematics we deal with only a very few aspects of the real world, namely numerical and geometric aspects of physical situations. As a student progresses through the curriculum, his mathematical concepts are refined, sharpened, and generalized to a much greater extent than his scientific ones, so the resulting mathematical conceptual structure differs markedly from one's scientific one. Hence, the process of concept formation in solving strategies may have quite different values in the two areas.

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[38]E.G. Begle, "Curriculum Research in Mathematics," The Journal of Experimental Education, Vol. XXXVII (Fall, 1968), pp. 44-48.

Estimation, Measurement, Intuition, and Plausibility of Answers

In mathematics the process of estimation of results and the acceptance of only plausible answers is very important. In the Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics[39] there is an extensive plan for the development of the important concept of inductive reasoning.

Faulk[40] explains that estimation is a process which is not new to children even in primary grades. They have made estimates of various types prior to the formal study of arithmetic. Teachers of elementary grades can help children to keep abreast of current developments in mathematics by teaching them to think quantitatively. Estimation of answers is one means toward that end. In considering this same end, Sims[41] stated that the practice of estimating a reasonable answer before performing the necessary computation will establish a criterion for plausible responses and help to discourage wild guesses. The estimate can establish feasible limits for working the problem and also can serve as a check on the final solution. Most word problems provide opportunity for practice of this skill.

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[39]National Council of Teachers of Mathematics, The Growth of Mathematical Ideas, K-12, The Twenty-Fourth Yearbook (Washington, D.C.: The Council, 1960), p. 183.

[40]Charles J. Faulk, "How Well Do Pupils Estimate Answers?" The Arithmetic Teacher, Vol. IX (December, 1962), pp. 436-440.

[41]Jacqueline Sims, "Improving Problem-Solving Skills," The Arithmetic Teacher, Vol. XVI (January, 1969), pp. 17-20.

To approach the main objective of problem solving, devising a plan, the pupil must not only understand meanings and identify available materials, but must also discern the relationships between them and intuitively select a plausible answer. Herlihy[42] conveys the fact that at this point one must consider the item of nonhabitual choice determining behavior. The teacher may guide the student at this difficult and critical point by asking questions which may help the student recall similar problems. Hopefully, this assistance may aid the student in discovering a workable pattern for his problem. The question may be restated, varied, and modified. To conceive an idea which will lead to the solution, the student will need formally acquired knowledge, good mental habits, concentration upon his purpose, and perserverence. It will be helpful for the student to learn to look for conditions in the problem rather than the "answer".

Many educators feel that measurement is an important part of the elementary child's mathematical experiences because a great many in-school and out-of-school activities involve measurement ideas. Even though number ideas are independent from measurement ideas, the use of measures can give concrete meaning to many number abstractions. If measurement is to be included in the curriculum of the elementary school, it is

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[42]Kathryn V. Herlihy, "A Look at Problem Solving in Elementary School Mathematics," The Arithmetic Teacher, Vol. XI (March, 1964), pp. 308-311.

necessary to know what facts the beginning elementary school child knows about measurement so that instruction can begin in appropriate places. Wilson and Cassell[43] convey the idea that the child should be able to estimate the dimensions of a room and should also be able to pace a specified distance with a fair degree of accuracy.

Davis et al[44] related that pre-school children have some understanding of common measures. Significant growth may occur between the nursery school and kindergarten years for some common measures. These findings lend encouragement to the belief that pre-school age children might profit from direct experiences designed to foster familarity with common measures and measurement. If subsequent research confirms hypotheses based on this belief, implications abound for arithmetic instruction in the early elementary school years. Certainly these present findings indicate that during the pre-school years of four and five, children have the beginnings of measurement and quantitative concepts.

With all the emphasis upon improving children's understanding of mathematics, Scott[45] reports, it is surprising that

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[43]Guy M. Wilson and Mabel Cassell, "A Research on Weights and Measures," The Journal of Educational Research, Vol. XLVI (April, 1953), pp. 575-585.

[44]O.L. Davis, Jr., Barbara Carper, and Carolyn Crigler, "The Growth of Pre-School Children's Familarity with Measurement," The Arithmetic Teacher, Vol. VI (October, 1959), pp. 189-190.

[45]Lloyd Scott, "A Study of the Case for Measurement in Elementary School Mathematics," School Science and Mathematics, Vol. LXVI (November, 1966), pp. 714-722.

greater concern has not been registered for the documented neglect of measurement and quantitative understanding at the elementary school level. Piaget has found that the general concepts of measurement to be attainable by children beyond approximately eight years of age. When the child appreciates that a linear segment may be conserved and that subdivision of the segment is possible without destruction of its totality, one is ready to learn measurement, according to Piaget.

Rosskopf[46] asserts that human beings tend to deal with classes of things instead of individuals in order to make some sense out of their environment. By forming such classes cognitive strain is reduced as well as the burden on memory. To form these classes, or categories, or sets, a person looks for cues, or, if you like, for characteristics that serve to distinguish things eligible for membership in the set from those that are not eligible. The point is that these categorizations are inventions, and this is particularly true in a complex body of knowledge like mathematics.

To solve a problem in mathematics, subsidiary concepts serve as mediators. Although initial focus and type of strategy have important roles, an individual will have little success in problem solving unless one has a good understanding of the mediating concepts. In order to achieve this end a student must develop his thought processes in such a manner that

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[46]Myron F. Roskopf, "Strategies for Concept Attainment in Mathematics," The Journal of Experimental Education, Vol. XXXVII (Fall, 1968), pp. 78-86.

he is intuitively capable of estimating the answers to varied problems and testing their plausibility. Considering the aforementioned process, Amir-Maez[47] says: "It is true that a student of mathematics must have a very sharp intuition, but he should never trust his intuition."

Finally, it is known that most subject-matter learning involves, as Scandura[48] injects, neither association nor concepts but, as they have been variously called by different investigators, "rules", "principles", "schemata", and "heuristics". This is even more true of mathematics learning than of learning in many other fields. To be more specific: "Meaningful learning implies the ability to give the appropriate response in a class or functionally distinct stimuli."

"Modern" mathematics has stressed the basic concepts of estimation, measurement, intuition, and the determination of the plausibility of results in problem solving situations. Yet, Newell[49] found that most pupils responded favorably and enthusiastically to some of the so-called new mathematics. Thus, each teacher has the responsibility for planning a program adequate for the times and adequate for the students.

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[47] Ali R. Amir-Maez, "Intuition and Mathematics," School Science and Mathematics, Vol. LXIV (December, 1964), pp. 767.

[48] Joseph M. Scandura, "Research in Psychomathematics," The Mathematics Teacher, Vol. LXI (October, 1968), pp. 581-591.

[49] Laura Newell, "Pupils Respond to the Modern Elementary Mathematics," The Arithmetic Teacher, Vol. XII (February, 1965), pp. 144-146.

The main goal is the achievement of the fullest development in mathematics for all students. This involves many factors: an interest and enthusiasm for mathematics, an understanding of the subject, an attitude of inquiry, an open mind, a willingness to explore and experiment, and a desire to make the subject interesting and appealing to preserve and strengthen the students' interest and enthusiasm.

Aspects of Mathematical Evaluation

Evaluation of mathematics programs must be in terms of the objectives indentified by each school. Each objective should be defined in terms of student performance so that it is possible to determine the extent to which the objectives have been reached.

Program evaluation should be continuous. Classroom teachers should cooperate in an effort to evaluate the results on instruction. This can often be done effectively through the construction and administration of school-wide or system-wide examinations. Such a procedure will help to indicate the effectiveness of both content and methods of instruction. It will also provide some indication of the extent to which individual needs are being met, and should help to indicate specific teaching needs for the immediate future.[50]

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[50] National Council of Teachers of Mathematics, Evaluation in Mathematics, The Twenty-Sixth Yearbook (Washington, D.C.: The Council, 1961), pp. 190-210.

In respect to this problem, Fey[51], stated:

> There is a realization that student achievement on some standardized test is a grossly inadequate measure for teaching success. More comprehensive diagnostic assessments of student outcomes must be developed and used.

Romberg and Wilson[52] suggested that commercially prepared standardized tests are not sufficient because:

> (1) They do not cover some important components of mathematical ability; (2) They tend to have been developed with the view of testing a unitary trait; (3) They tend to 'over-test' some components; (4) They often include items for testing components of little importance; (5) They are too long, from one to three hours; and (6) They are primarily for the assessment of individuals rather than groups.

Bruecker[53] reports that there exists tests which have recently been devised to measure understandings of the structure of number systems and skill in interpreting graphs. Yet, he maintains that these measures are most difficult to assess. Included in this categorization was the ability to apply quantitative procedures and methods of thinking in social situations. Von Brock[54] mentions that:

> One of the difficulties in evaluating a child's growth in arithmetic has been that most instruments are based on computational skills, the assumption being that, if a child

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[51]James Fey, "Classroom Teaching of Mathematics," Review of Educational Research, Vol. XXXIX (October, 1969), p. 548.

[52]Thomas A. Romberg and James W. Wilson, "The Development of Mathematics Achievement Tests for the National Longitudinal Study of Mathematical Abilities," The Mathematics Teacher, Vol. LXI (May, 1968), pp. 489-495.

[53]Leo J. Bruecker, "Evaluation in Arithmetic," Education, Vol. LXXIX (January, 1959), p. 292.

[54]Robert Von Brock, "Measuring Arithmetic Objectives," The Arithmetic Teacher, Vol. XII (November, 1965), p. 537.

> could manipulate numbers through the basic processes, he understood arithmetic. This assumption was probably adequate a generation ago, but our concern today is more with the understanding of the process than with mere computation. The ability to comprehend, apply, and analyze arithmetic concepts has become a major factor in developing the elementary school arithmetic program of today.
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> To assist the individual child and to evaluate existing and future programs in arithmetic, we need to consider how these more abstract objectives might be measured.

Further substantiation of the importance of this type of testing was confirmed by Shane[55] who concluded that in order to develop facility in quantitative reasoning there must exist constant evaluation of mathematical objectives and student progress.

The need for continuous evaluation of the effectiveness of a mathematics curriculum is promoted by Johnson[56]. Such evaluation can be made only partially by tests and examinations. The performance of the student outside the classroom is significant in evaluating achievement. For classroom evaluation, he suggested, achievement should be measured in terms of:

> (1) All the goals in the curriculum; (2) Growth, change, and progress in the attainment of goals; (3) Ability to use the facts, skills, and principles learned; (4) Retention over a long period of time; (5) Levels of mastery and understanding.

The article continues:

> To measure the attainment of these standards, it will be necessary to use a variety of tests including reading,

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[55] Harold G. Shane and E.T. McSwain, Evaluation and the Elementary Curriculum (New York: Henry Holt and Company, Inc., 1958), p. 227.

[56] Donovan A. Johnson, "Next Steps in Secondary School Mathematics," Theory into Practice, Vol. III (April,1964), pp. 71-75.

> problem solving performance, and open-book tests, as well as achievement tests. The evaluation should include observations, inventories, check lists, appraisal of student products, and the use of teacher made tests to supplement published tests. The results should be interpreted carefully in terms of the group involved and should then be used to evaluate the effectiveness of the particular mathematics program.

Sueltz[57] contends that measurement of understanding and judgment in mathematics is possible. Three methods consisting of objective materials, pictorial and illustrated materials combined with verbal statements, and normal school experiences may be used. In each of these methods, emphasis must be placed upon the role of the teacher and the close tie between measurement and learning.

In respect to the evaluation of quantitative understanding Muscio[58] writes the following:

> It would appear that the high achiever on a measure of quantitative understanding may best be characterized by (a) his superior intellectual capacity, (b) his greater verbal ability as extended to all areas of learning, and (c) his superior achievement in the areas of reading and arithmetic, even when the effects of mantal age have been discounted. In addition, he may be expected to express a somewhat more favorable attitude toward mathematics than does the low achiever, and he is likely to be somewhat younger than his classmates. Finally, there is a better than even chance that the high achiever will be a boy and, on the basis of previous findings, will come from a somewhat higher than average socioeconomic neighborhood. (used <u>Functional Evaluation in Mathematics, Test 1: Quantitative Understanding</u> by B.A. Sueltz)

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[57]Ben A. Sueltz, "The Measurement of Understanding and Judgments in Elementary School Mathematics," <u>The Mathematics Teacher</u>, Vol. XL (October, 1947), pp. 279-284.

[58]Robert D. Muscio, "Factors Related to Quantitative Understanding in Sixth Grade," <u>The Arithmetic Teacher</u>, Vol. IX (May, 1962), pp. 258-262.

Madden[59] stresses that mathematics is a language and a way of thinking. Evaluation of mathematical competence should include measures which test the student's understanding of the meaning of number, of volume, and of sequences. Of greater importance is measurement of the tendency and the ability to think as mathematical people think, to define the problems precisely, to state them succinctly, and to relate them quantitatively to whatever is appropriate.

Eads[60] contends that:

> What pupils learn in school is largely determined by what teachers and parents really hold to be of value. Where quantity and speed of learning in mathematics are of primary importance, pupils will gain approbation for uncertain memoriter-type learning. Few pupils will then attach importance to discovering mathematical relationships, thinking out solutions in original ways, computing "mentally" whenever possible, making estimates before written computation, etc. Teachers also reflect the values of their teachers and supervisors. Where instruction is viewed in terms of pupil performance on tests administered centrally, teachers will emphasize speed and accuracy on paper-andpencil tests and will try to bring all students "up to grade norm". Few teachers will then attach importance to activities planned to teach mathematical meanings or designed to encourage all pupils to do mathematical thinking at their respective levels of ability.

Although it is still too early to present reliable test results to substantiate, or disprove, the claims made for modern mathematics programs, Hipwood[61] found that in terms of

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[59] Madden, "New Directions in the Measurement of Mathematical Ability," pp. 375-379.

[60] Laura K. Eads, "Evaluation of Learning in Arithmetic," The Bulletin of the National Association of Secondary School Principals, Vol. XLIII (May, 1959), pp. 128-130.

[61] Stanley J. Hipwood, "Modern Mathematics--Go or No Go?" The Arithmetic Teacher, Vol. XII (February, 1965), pp. 120-122.

average pupil gain, that apparently modern mathematics programs promote better pupil performance in quantitative thinking and abstract reasoning as measured by the Iowa Tests of Educational Development.

A problem of urgent dimensions facing education today is the identification and encouragement of above average and exceptional students. MacKinnon[62] relates that:

> Our task as educators is not to recognize creative talent after it has come to expression, but either through our insight or through the use of validated predictors to discover talent when it is still latent and to provide that kind of educational climate and environment which will facilitate its development and expression.

Finally, Suppes[63] makes the following pertinent points:

> The ability to learn the new mathematics concepts is not restricted to gifted children. I have attempted to show how rapidly gifted children advance through the new concepts organized in terms of the standard curriculum aimed at the average child. Probably the most important point to draw from our own research is that brighter children--those, let us say, in the upper quartile--are able to cover a great deal many more mathematical concepts and to learn a good deal more mathematics during their elementary years than we have been willing to admit in the recent past.

Some Relationships of Quantitative Judgment to Other Factors

Johnson[64] concluded that knowledge of vocabulary is

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[62]Donald W. MacKinnon, "Identifying and Developing Creativity," The Journal of Secondary Education, Vol. XXXVIII (March, 1963), p. 166.

[63]Patrick Suppes, "The Ability of Elementary School Children to Learn the New Mathematics," Theory into Practice, Vol. III (April, 1964), pp. 57-61.

[64]John T. Johnson, "On the Nature of Problem Solving in Arithmetic," The Journal of Educational Research, Vol. XLIII (October, 1949), pp. 110-115

more important than reasoning ability when considering success in problem solving.

Christantiello[65] writes that many persons concerned with mathematics learning believe that "non-intellectual" factors such as attitude and emotional makeup have an important bearing upon a student's success with a subject.

Anecdotal evidence from teachers, injects Suppes[66], suggest that there exists some carry-over in critical thinking and attitude into other fields, especially arithmetic, reading, and English. More explicit behavorial data on carry-over in critical thinking would be desirable.

Extensive research on the relationship between reading skills and mathematical problem solving ability has been conducted by Treacy[67]. He concluded that high achievers in problem solving are significantly superior to low achievers in reading skills and intellectual ability. Similarly, high achievers obtain scores which are significantly higher at the one percent level in quantitative relationships.

General reading ability, Balow[68] notes, does have an

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[65]Philip D. Christantiello, "Attitude Toward Mathematics and the Predictive Validity of a Measure of Quantitative Aptitude," The Journal of Educational Research, Vol. LXI (June, 1968), p. 184.

[66]Patrick Suppes and Frederick Binford, "Experimental Teaching of Mathematical Logic in the Elementary School," The Arithmetic Teacher, Vol. XII (March, 1965), pp. 187-195.

[67]John P. Treacy, "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," The Journal of Educational Research, Vol. XLVIII (September 1944-May 1945), p. 92.

[68]Irving Balow, "Reading and Computational Ability as Determinants of Problem Solving," The Arithmetic Teacher, Vol. XI (January, 1964), pp. 18-22.

effect on problem solving ability when the total range of reading ability was used and the effect of intelligence was controlled. Findings indicated that it is important to consider children's reading ability as well as computational ability when teaching problem solving skills involving quantitative terms.

Muscio[69] found that boys were significantly superior to girls in aspects of quantitative understanding. Superiority was attributable to neither general intelligence nor computational skills. He also found positive correlations between scores on quantitative understanding tests and achievement in arithmetic computation, arithmetic reasoning, and mathematical vocabulary. Further correlations were found to exist between quantitative understanding, general reading ability, and intellectual capacity. In addition, the student's attitude toward mathematics was not an adequante predictor of ability in quantitative understanding.

Wozencraft[70] found just the opposite sex variation in aspects of quantitative understanding than did Muscio. Her results found the girls scoring significantly superior to the boys.

Wick[71] found that the high school mathematics record

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[69] Muscio, "Factors Related to Quantitative Understanding in Sixth Grade," pp. 261-262.

[70] Marian Wozencraft, "Are Boys Better Than Girls in Arithmetic?" The Arithmetic Teacher, Vol. X (December, 1963), pp. 489-490.

[71] Marshall E. Wick, "A Study of the Factors Associated with Success in First-Year College Mathematics," The Mathematics Teacher, Vol. LVIII (November, 1965), pp. 647-648.

was one of the best predictors of success in first year college mathematics. In the same type of study, Barnes and Asher[72] noted that the best predictor of ninth grade algebra one grades was the eighth grade mathematics score.

Thompson[73] reported that the effects of readability and mental ability on arithmetic problem solving performances were interactive. Although ease of reading was associated with higher performance at both high and low levels of mental ability, the effect was greater with subjects of low mental ability.

To what extent the teaching of reading and the teaching of mathematics are similar or can be similar is not yet determined. Fitzgerald[74] asserts there is certainly more more material available for teaching reading; but recent trends in research and publications, such as enrichment topics, teaching machines, etc., indicate an increase of materials which would allow an individual to learn mathematics independently.

The age at which children can profitably be exposed to sound mathematical ideas was once thought to be known fairly conclusively, but this idea has recently been exploded by numerous counterexamples. In fact, grade placement of topics

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[73] Elton N. Thompson, "Readability and Accessory Remarks: Factors in Problem Solving in Arithmetic," Ph.D. Thesis. Stanford University, 1967. Dissertation Abstracts 28: 2464A; No. 7, 1968.

[74] William M. Fitzgerald, "On Learning of Mathematics by Children," The Mathematics Teacher, Vol. LVI (November, 1963), pp. 517-521.

has progressed to the point that Bruner[75] now claims: "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." In short, we still have a very limited knowledge about which children can learn which ideas at which stage in their development.

Unkel[76] states that:

> Socioeconomic status has a significant effect on achievement in arithmetic even when based on the difference between a pupil's actual achievement score in arithmetic and his presumed potential (chronological age, grade placement, and his score on a test of mental age). That is, in general it can be said that the achievement of children of comparable mental ability is affected by socioeconomic status, with pupils in high socioeconomic groups attaining the highest achievement level, and students in the middle groups attaining the next level, and students in the low groups having the lowest level of achievement.
>
> The single exception was in arithmetic reasoning, where pupil scores showed no significant difference between the middle and high socioeconomic groups.

The Development of Mathematical Abilities

One of the phenomena of the past decade, reported by Romberg[77], has been the intellectual stimulation and volume of research studies generated by the observations and theories of Jean Piaget. Piaget's vast conceptualization of human cognitive

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[75] Jerome S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1961), p. 33.

[76] Esther Unkel, "A Study of the Interaction of Socioeconomic Groups and Sex Factors with the Discrepancy Between Anticipated Achievement and Actual Achievement in Elementary School Mathematics," The Arithmetic Teacher, Vol. XIII (December, 1966), pp. 662-670.

[77] Thomas A. Romberg, "Current Research in Mathematics Education," Review of Educational Research, Vol. XXXIX (October, 1969), pp. 480-481.

development is of particular importance to mathematics educators since most of his observations have been on mathematical tasks (quantification, geometry, spatial relations, logic, etc.).

Coxford[78] maintains:

> A developmental approach appears to be the best for instruction in number and measurement. That is, the learner is helped to discover the concepts of measurement and number through experiences that not only bear directly on the concept in question but also are appropriate to the child's individual growth and development. This is indicated by all of Piaget's work. The concepts of number and measurement develop through time, and thus instruction can be geared to this developmental process.

Piaget and experienced teachers may disagree on the age at which any precise understanding of mathematics, science, logical argument, and scientific attitudes are possible. Newbury[79] suggests that the long-term aims should certainly include the testing of hypotheses and the formation of general principles. Furthermore, he feels that it seems to be useful to stress to children the value of the quantitative approach in solving problems. As an essential part of their training, the children should individually guess the result before the activity is carried out. The checking of the answer by some precise form of measurement and the use of suitable units en-

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[79]N.F. Newbury, "Quantitative Aspects of Science at the Primary Stage," The Arithmetic Teacher, Vol. XIV (December, 1967), pp. 641-644.

ables a child to find out how accurate his guess or estimation was. Using worthwhile problems, appropriate ways of measuring the degree of accuracy should gradually be introduced to the student who is being taught to reason and think in a quantitative manner.

CHAPTER III

PROCEDURE

General Plan of Procedure

Initially, the writer familiarized himself with the Test of Quantitative Judgment, Form T, and the results obtained by Hall[1] and Tuttle[2].

The problem of this study involved the following parts: (1) To measure the ability of upper elementary and junior high school students on aspects of quantitative judgments relative to their normal social environments; (2) To determine the relationships between students' abilities in dealing with quantitative judgments in relation to their sex, Iowa Achievement Scores, intelligence quotients, grade levels, and their mathematics grades for the previous two years; (3) To determine if there exists an important amount of variance remaining after removing the variation contributed by the other measures in the study (I.Q. test, Iowa tests, grade level, and previous mathematics grades).

A sample of 884 students was selected from grades six through nine in a small Western Massachusetts city. The sample

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[1]Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment," (unpublished Ed.D. dissertation, School of Education, Boston University, 1965).

[2]Cynthia L. Tuttle, "The Refinement of a Test of Quantitative Judgment," (unpublished M.Ed. thesis, School of Education, University of Massachusetts, 1965).

as previously mentioned consisted of students from four elementary schools and one junior high school. The schools and classrooms were selected on the basis of three factors: (1) They contained a mixture of average and above-average students; (2) The personnel involved were willing to participate in the study; and (3) It was possible to obtain at least 200 students per grade level.

The Test of Quantitative Judgment was administered twice. At least 85 percent or more per grade level took the second administration of the test precisely fourteen days after the first administration in an attempt to estimate a measure of reliability for the Test of Quantitative Judgment. Winer's method[3] of estimating reliability using the analysis of variance format was used in determining the reliability of the Test of Quantitative Judgment.

Data Needed

For the purpose of the study it was necessary to collect data on several specific variables that would be considered in the study. A score or index was needed on each student in the study as to sex, grade, chronological age, intelligence quotient, tests of fundamental skills, previous mathematics

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[3]B.J. Winer, Statistical Principles in Experimental Design, (New York: McGraw-Hill Book Company, 1962), pp. 124-132.

grades for the preceding two years, and quantitative judgment.

The California Test of Mental Maturity (Short Form) was selected and administered to each student in the sample. "The CTMM[4] is a group test for measuring mental capacity. It reveals information that is basic to any interpretation of present functioning and future potential in a relatively specific but critical area of human activities."

Furthermore, the investigator wanted to determine whether or not the Test of Quantitative Judgment was measuring a unique factor and not replicating standardized tests. For this purpose The Iowa Tests of Basic Skills were used. The ITBS were chosen because they provided comprehensive measurement in the following areas:[5] "Vocabulary, reading, the mechanics of correct writing, methods of study, and arithmetic." The investigator used the results of four of these tests: Vocabulary, Reading Comprehension, Arithmetic Concepts, and Arithmetic Reasoning.

Instrumentation

The data for the study was obtained in the following manner: (1) Sex, age, grade level, and the previous two years

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[4] E.T. Sullivan, Willis W. Clark, and Ernest W. Tiegs, Manual for the California Short Form Test of Mental Maturity, (Los Angles: California Test Bureau, 1957), p. 2.

[5] Teacher's Manual for The Iowa Tests of Basic Skills, (Boston: Houghton Mifflin Company, 1964), p. 3.

mathematics grades of each student in the study were acquired by the investigator from the school records; (2) Intelligence quotients were procured by the use of the California Test of Mental Maturity; (3) Scores of achievement were determined using the Iowa Tests of Basic Skills; and (4) Scores of quantitative judgment were secured using Hall's Test of Quantitative Judgment, Form T.

Test Administration

The investigator prepared for the most uniform administration of the Test of Quantitative Judgment by utilizing the following techniques: (1) All instructions were written on the first page of the test booklet; (2) Each student used IBM answer cards with mark sense pencils; (3) The investigator or teacher administering the test read and explained the instructions on the first page of the test booklet; (4) The investigator met with all assisting personnel in order to establish uniform guidelines for administering the Test of Quantitative Judgment; (5) Each student's answer card was checked by the examiner to verify that the student had properly followed the initial instructions; and (6) Each class was instructed in the appropriate method of entering the responses on the IBM answer card.

After the tests were administered, all completed answer cards were inspected by the investigator to eliminate extraneous marks and responses containing more than one answer.

Following this step, the cards were machine scored using an IBM 1620 Computer. After correcting both the initial Test of Quantitative Judgment and the retest of the Test of Quantitative Judgment an item analysis was carried out by the IBM 1620 Computer on the results of the first administration of the Test of Quantitative Judgment. Chung-Teh Fan's Item Analysis Table was used to interpret these results.

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[6] Chung-Teh Fan, Item Analysis Tables, (Princeton: Educational Testing Service, 1952).

CHAPTER IV

ANALYSIS OF DATA

The purpose of this study was to determine the following:

(1) Is there an important amount of variance remaining after removing the variation accounted for by the following:

Q.J. - (Variance of the California Test of Mental Maturity)

Q.J. - (Variance of parts of the Iowa Tests of Basic Skills)(Vocabulary, Reading, Arithmetic Concepts, Arithmetic Reasoning)

Q.J. - (Variance contributed by previous mathematics achievement grades for the previous two years)?

(The following are null hypotheses)

(2) There is no relationship between children's ability to deal with aspects of quantitative judgment and their grade level (age).

(3) There is no relationship between children's ability to deal with aspects of quantitative judgment and their sex.

(4) There is no relationship between children's ability to deal with aspects of quantitative judgment and their intelligence quotient.

The scores on the Test of Quantitative Judgment used in this study were analyzed for measures of central tendency, variability, and reliability. These results are reported in Tables 2 and 3.

Means and Standard Deviations

The means and standard deviations for the Test of Quantitative Judgment are reported for all grades in the sample. The analysis consists of computations by sex as well as the total subsample in each grade.

There exists a difference in the means between males and females within each grade level. The differences range from 0.85 units at the sixth grade level to 2.61 units at the eighth grade level. In all cases, the males have a higher mean on the Test of Quantitative Judgment. The differences between standard deviations for both sexes in grades six and seven were greater than the differences noted in grades eight and nine. (ie., with increases in grade the scores tended to less dispersed about their mean.) There was an increase in the standard deviation from grade six to grade seven. After this increase the standard deviation decreases between the seventh, eighth, and ninth grade levels. The increase in the means of the Test of Quantitative Judgment from grade level to grade level suggests that the difference in difficulty is by grade level. The differences in the means between sexes possibly implies that the males perform observably higher than the females on the aspects of quantitative judgment used in this study. Furthermore, these results of the Test of Quantitative Judgment seem to indicate that quantitative judgment may be, at least in part, a learned phenomenon. (The gain scores and decreases in standard deviation are occurring with increases in grade level.)

Reliability Coefficients

Reliability of the Test of Quantitative Judgment was determined using Winer's[1] Analysis of Variance Method to Estimate Reliability of Measurements. These results are reported in Table 3.

$$\text{Reliability} = 1 - \frac{MS_{\text{within people}}}{MS_{\text{between people}}}$$

$$MS_{\text{within people}} = \frac{SS_{\text{within people}}}{n}$$

$$MS_{\text{between people}} = \frac{SS_{\text{between people}}}{n - 1}$$

n = number of people

$$SS_{\text{within people}} = \sum_{i=1}^{n} \left(\sum_{j=1}^{2} X_{ij}^2 \right) - \frac{\sum_{i=1}^{n} P_i^2}{k}$$

X_{ij} = score of the i^{th} person on the j^{th} administration of the test of Quantitative Judgment

P_i = Sum of QJT and QJR scores of the i^{th} individual where QJT and QJR are the first and second administrations of the Test of Quantitative Judgment respectively.

$k = 2$ (number of administrations of QJ)

$$SS_{\text{between people}} = \frac{\sum_{i=1}^{n} P_i^2}{k} - \frac{G^2}{kn}$$

$$G = \sum_{i=1}^{n} P_i$$

The reliability coefficients seem adequate since the overall

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[1]B.J. Winer, Statistical Principles in Experimental Design (New York: McGraw-Hill, 1962), pp. 124-132.

reliability coefficient is 0.87. Yet, one must note that there is a rather large decrease in the reliability coefficients for grade nine. This would suggest that this form of the Test of Quantitative Judgment is probably better suited for grades six, seven, and eight.

On the initial Test of Quantitative Judgment, Hall[2] gave the following reliability coefficients:

Using the Kuder-Richardson Formula 20:

Grade 4 (.77), Grade 5 (.67), Grade 6 (.72)

Using Pearson's Coefficient of Correlation:

Grade 4	(.78),	Grade 5	(.82),	Grade 6	(.87)
Male	(.81),	Male	(.86),	Male	(.84)
Female	(.77),	Female	(.78),	Female	(.90)

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[2]Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment," (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), pp. 59-60.

TABLE 2

STATISTICAL ANALYSIS OF THE TEST OF QUANTITATIVE JUDGMENT FOR GRADES 6, 7, 8, and 9

	MALE			FEMALE			TOTAL		
Grade	N	Mean	S.D.	N	Mean	S.D.	N	Mean	S.D.
Six	132	37.32	6.26	117	36.47	5.54	249	36.92	5.94
Seven	101	38.74	6.93	109	37.72	5.91	210	38.21	6.43
Eight	104	41.91	5.42	104	39.30	5.62	208	40.61	5.63
Nine	97	42.67	4.98	120	40.58	4.84	217	41.52	5.00

TABLE 3

RELIABILITY COEFFICIENTS USING WINER'S METHOD

	N	MALE	N	FEMALE	N	TOTAL
Grade 6	119	.83	108	.82	227	.83
Grade 7	91	.85	101	.90	192	.88
Grade 8	91	.95	94	.99	185	.96
Grade 9	87	.64	108	.72	195	.69
Total	388	.82	411	.94	799	.87

Norms

Rank order norms based on percentages of students receiving a specified raw score were compiled and are reported by grade.

Frequency polygons of the scores on the Test of Quantitative Judgment by grade and sex are displayed. Furthermore, cumulative frequency ogives by grade level are included.

RANK ORDER NORMS BASED ON GRADE SIX RAW SCORES

Raw Score	N 249 Grade 6 Total	%	N 132 Grade 6 Male	%	N 117 Grade 6 Female	%
52						
51	1	100			1	100
50	0	98.82			0	98.93
49	1	98.82	1	100	0	98.93
48	0	98.42	0	98.40	0	98.93
47	6	98.42	4	98.40	2	98.93
46	4	95.40	2	95.40	2	97.15
45	5	94.40	2	93.95	3	95.35
44	8	92.40	6	92.45	2	93.82
43	16	89.20	12	87.90	4	92.04
42	16	82.75	9	78.80	7	88.64
41	14	76.30	7	72.00	7	82.64
40	21	70.65	15	66.70	6	76.64
39	13	62.20	6	55.40	7	71.64
38	17	57.00	10	50.85	7	65.64
37	24	50.15	11	43.35	13	59.64
36	15	41.30	6	35.05	9	48.04
35	17	35.25	10	30.50	7	40.34
34	10	28.40	1	23.30	9	34.34
33	11	24.40	6	22.55	5	26.64
32	10	20.00	3	18.00	7	22.39
31	8	16.00	3	15.75	5	16.39
30	5	12.80	3	13.50	2	12.14
29	3	10.80	1	11.25	2	10.36
28	5	9.60	2	10.50	3	8.58
27	3	7.60	2	9.00	1	6.03
26	2	6.40	1	7.50	1	5.18
25	3	5.60	1	6.75	2	4.33
24	3	4.40	3	6.00	0	2.55
23	3	3.20	2	3.75	1	2.55
22	2	2.00	1	2.25	1	1.70
21	1	1.20	1	1.50	0	.85
20	1	.80	0	.75	1	.85
19	0	.40	0	.75		
18	0	.40	0	.75		
17	0	.40	0	.75		
16	0	.40	0	.75		
15	0	.40	0	.75		
14	1	.40	1	.75		
13						

RANK ORDER NORMS BASED ON GRADE SEVEN RAW SCORES

Raw Score	N 210 Grade 7 Total	%	N 101 Grade 7 Male	%	N 109 Grade 7 Female	%
51						
50	2	100	2	100		
49	1	98.92	1	97.80		
48	4	98.45	3	96.81	1	100
47	7	96.55	4	93.86	3	98.75
46	3	93.22	2	89.91	1	96.00
45	10	91.80	8	87.93	2	95.10
44	11	87.04	6	80.03	5	93.30
43	16	81.81	5	74.09	11	88.75
42	11	74.19	5	69.14	6	78.65
41	15	68.96	5	64.19	10	73.15
40	15	61.82	7	59.24	8	64.00
39	19	54.68	7	52.32	12	56.70
38	19	45.63	13	45.40	6	45.70
37	15	36.58	7	32.60	8	40.20
36	5	29.44	1	25.68	4	32.90
35	8	27.06	5	24.69	3	29.25
34	11	23.25	4	19.74	7	26.50
33	6	18.02	1	15.79	5	20.10
32	5	15.16	3	14.80	2	15.55
31	6	12.78	2	11.85	4	13.75
30	5	9.92	3	9.87	2	10.10
29	3	7.54	0	6.92	3	8.25
28	1	6.12	0	6.92	1	5.50
27	1	5.65	0	6.92	1	4.55
26	1	5.18	1	6.92	0	3.65
25	1	4.71	1	5.94	0	3.65
24	1	4.24	0	4.95	1	3.65
23	0	3.77	0	4.95	0	2.75
22	1	3.77	0	4.95	1	2.75
21	1	3.30	1	4.95	0	1.80
20	1	2.83	1	3.95	0	1.80
19	2	2.36	1	2.95	1	1.80
18	0	1.41	0	1.98	0	.90
17	0	1.41	0	1.98	0	.90
16	1	1.41	1	1.98	0	.90
15	1	.94	1	.99	0	.90
14	0	.47	0		0	.90
13	1	.47	0		1	.90
12						

RANK ORDER NORMS BASED ON GRADE EIGHT RAW SCORES

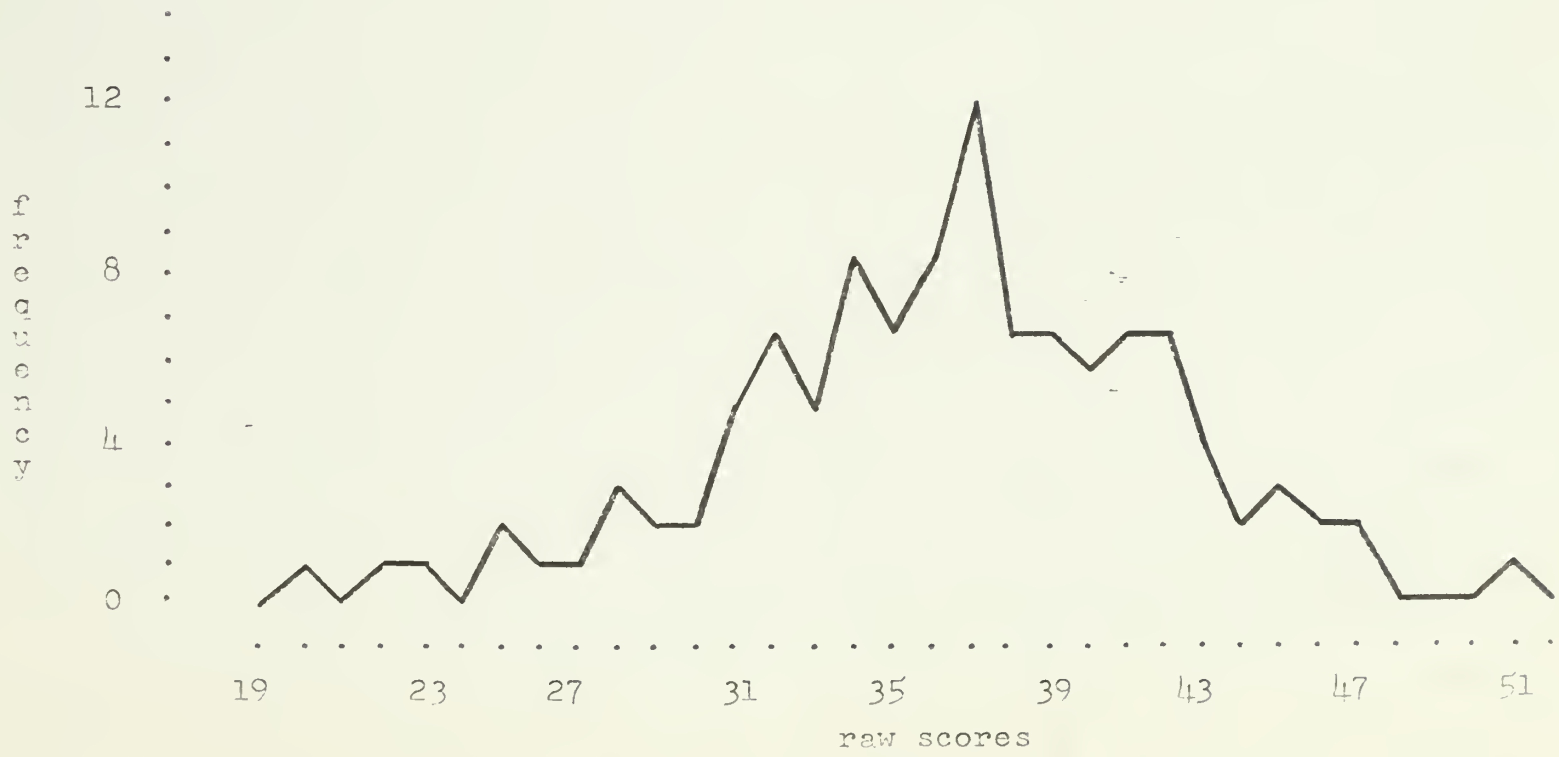
Raw Score	N 208 Grade 8 Total	%	N 104 Grade 8 Male	%	N 104 Grade 8 Female	%
53						
52	2	100	1	100	1	100
51	3	98.95	1	98.93	2	98.94
50	5	97.51	4	97.97	1	97.02
49	4	95.11	4	94.13	0	96.06
48	4	93.19	3	90.29	1	96.06
47	7	91.21	6	87.41	1	95.10
46	16	87.29	11	81.65	5	94.14
45	15	80.22	9	71.08	6	89.34
44	15	73.01	7	62.43	8	83.58
43	15	65.80	7	55.70	8	75.89
42	15	58.59	6	48.97	9	68.20
41	14	51.38	9	43.21	5	59.55
40	11	44.65	5	34.56	6	54.75
39	11	39.37	5	29.76	6	48.99
38	13	34.09	5	24.96	8	43.23
37	8	27.84	5	20.16	3	35.54
36	11	24.00	3	15.36	8	32.66
35	9	18.72	4	12.48	5	24.97
34	8	14.40	0	8.64	8	20.17
33	8	10.56	5	8.64	3	12.48
32	2	6.72	0	3.84	2	9.60
31	2	5.76	0	3.84	2	7.68
30	1	4.80	1	3.84	0	5.76
29	1	4.32	0	2.88	1	5.76
28	1	3.84	1	2.88	0	4.80
27	5	3.36	1	1.92	4	4.80
26	0	.96	0	.96	0	.96
25	1	.96	1	.96	0	.96
24	0	.48	0		0	.96
23	1	.48	0		1	.96
22						

RANK ORDER NORMS BASED ON GRADE NINE RAW SCORES

Raw Score	N 217 Grade 9 Total	%	N 97 Grade 9 Male	%	N 120 Grade 9 Female	%
53						
52	2	100	2	100		
51	2	98.52	2	97.86		
50	3	97.60	1	95.80	2	100
49	7	96.22	6	94.77	1	98.29
48	7	93.00	4	88.59	3	97.46
47	7	89.78	4	84.47	3	94.96
46	13	86.56	8	80.35	5	92.46
45	22	80.57	10	72.11	12	88.30
44	16	70.44	7	61.81	9	78.30
43	19	63.07	7	54.60	12	70.80
42	20	54.32	8	47.39	12	60.80
41	23	45.11	11	39.15	14	50.80
40	9	34.51	6	27.81	3	39.14
39	13	30.37	7	21.63	6	36.64
38	8	24.38	1	14.42	7	31.64
37	10	20.24	6	13.39	4	25.81
36	8	15.64	2	7.21	6	22.48
35	9	11.96	2	5.15	7	17.48
34	5	7.82	0	3.09	5	11.65
33	2	5.52	1	3.09	1	7.49
32	4	4.60	0	2.06	4	6.66
31	0	2.76	0	2.06	0	3.33
30	1	2.76	0	2.06	1	3.33
29	1	2.30	0	2.06	1	2.50
28	1	1.84	0	2.06	1	1.66
27	0	1.38	0	2.06	0	.83
26	1	1.38	1	2.06	0	.83
25	0	.92	0	1.03	0	.83
24	1	.92	0	1.03	1	.83
23	0	.46	0	1.03		
22	0	.46	0	1.03		
21	1	.46	1	1.03		
20						

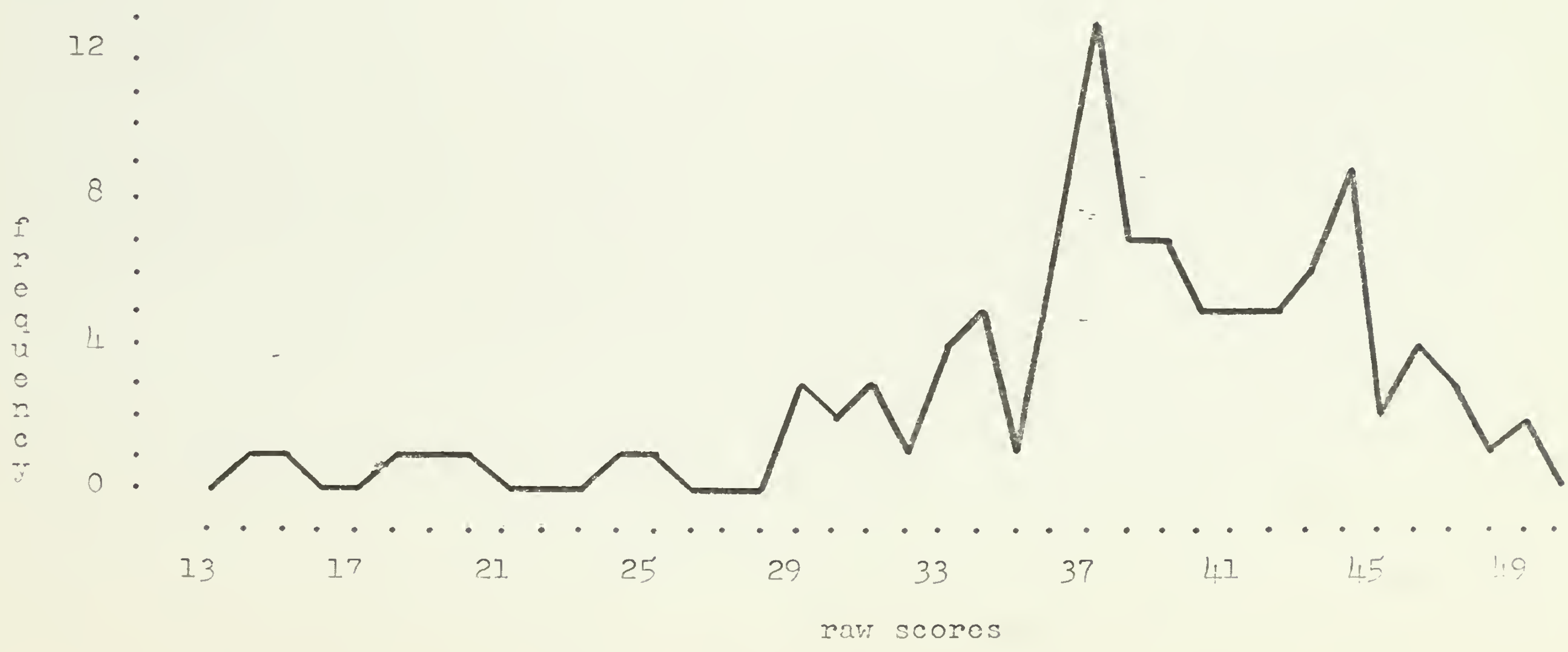
FREQUENCY POLYGON OF SIXTH GRADE MALE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT

FREQUENCY POLYGON OF SIXTH GRADE FEMALE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT

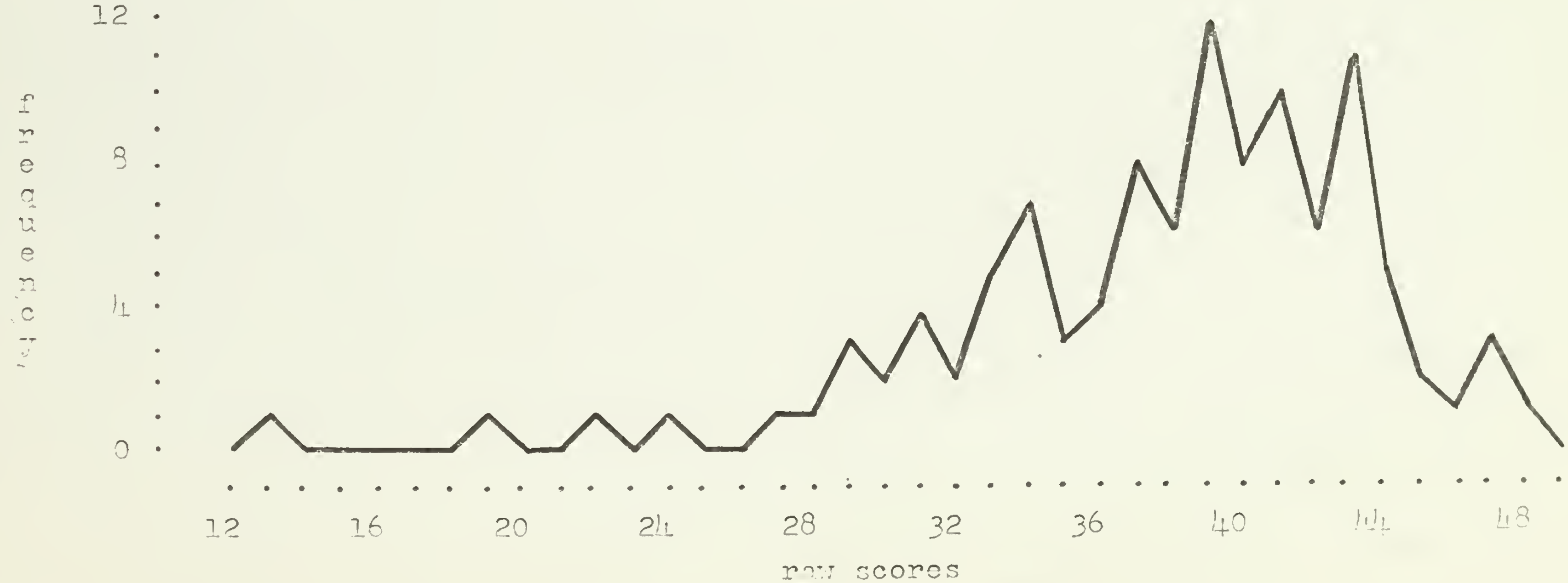


FREQUENCY POLYGON OF TOTAL SIXTH GRADE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT

FREQUENCY POLYGON OF SEVENTH GRADE MALE RAW SCORES ON TEST OF QUANTITATIVE JUDGMENT



FREQUENCY POLYGON OF SEVENTH GRADE FEMALE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT



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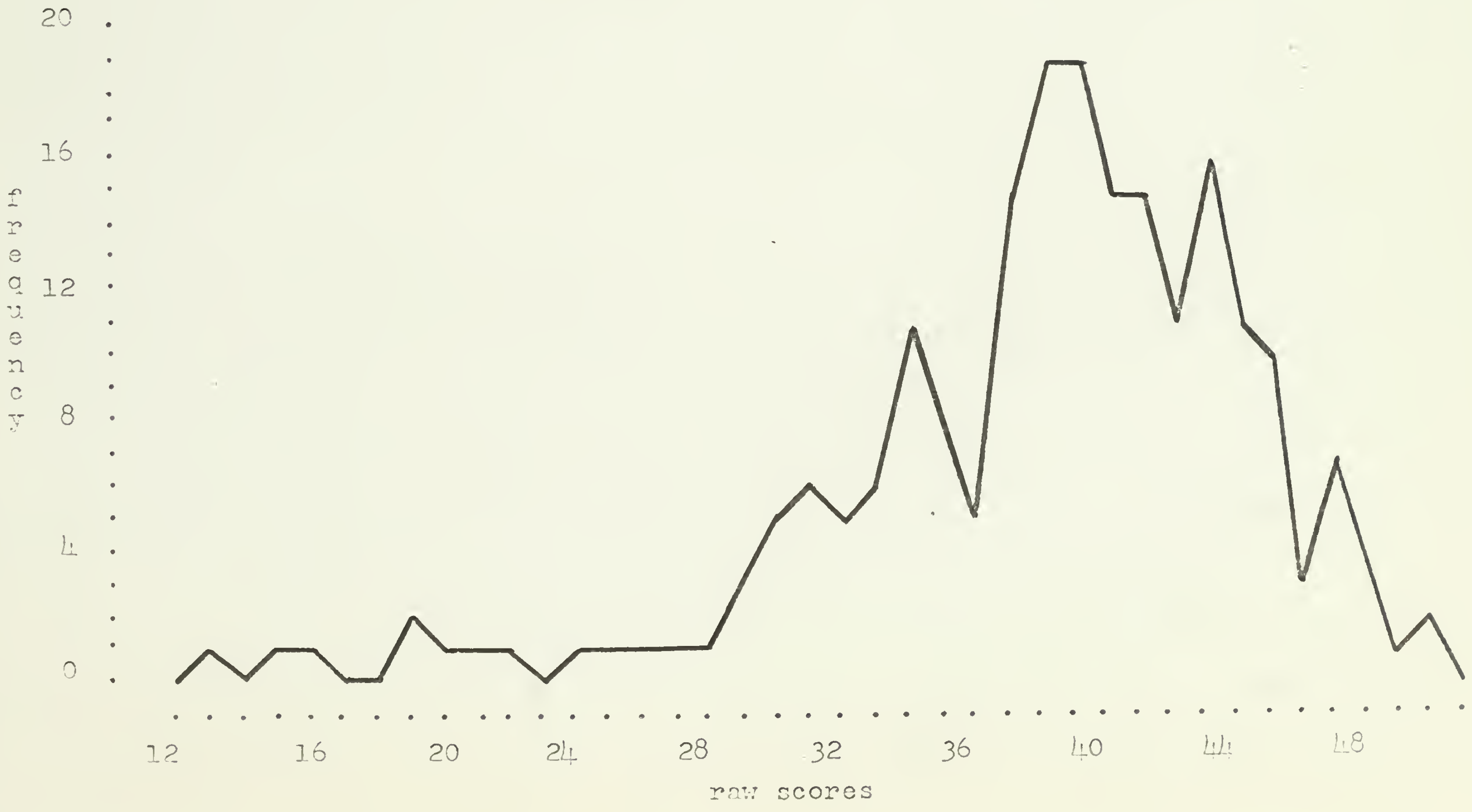
ON TEST OF QUANTITATIVE JUDGMENT



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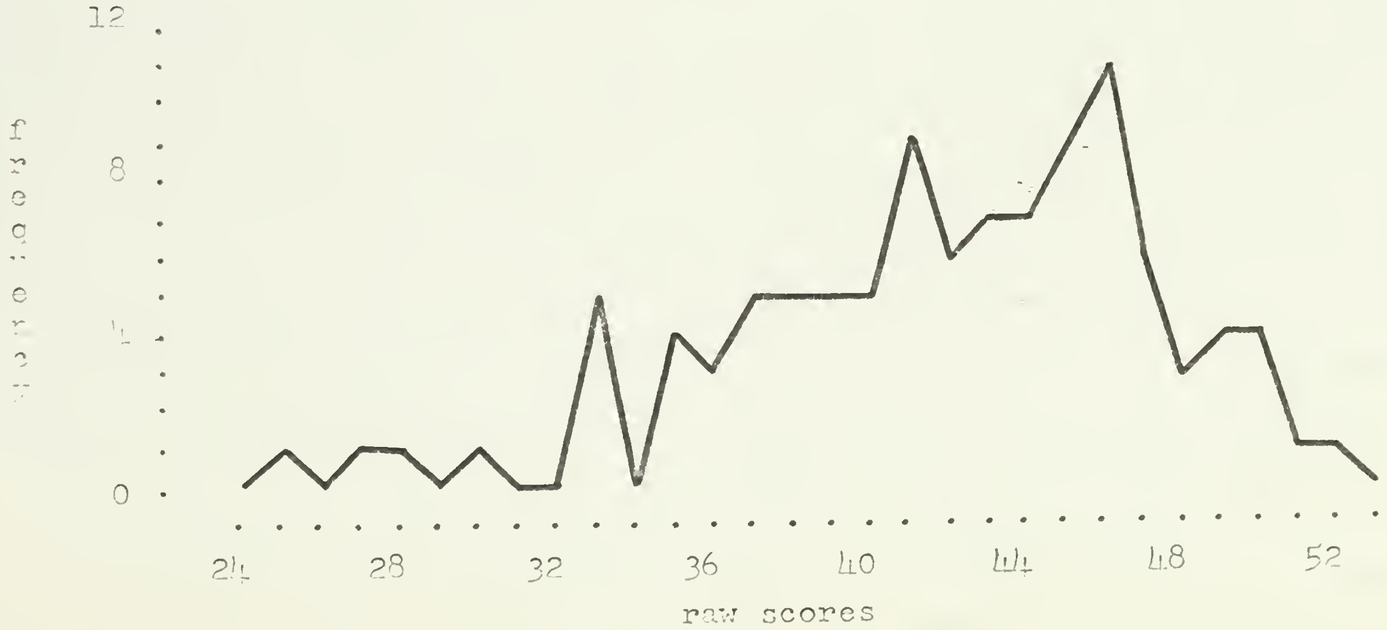
ON TEST OF QUANTITATIVE JUDGMENT



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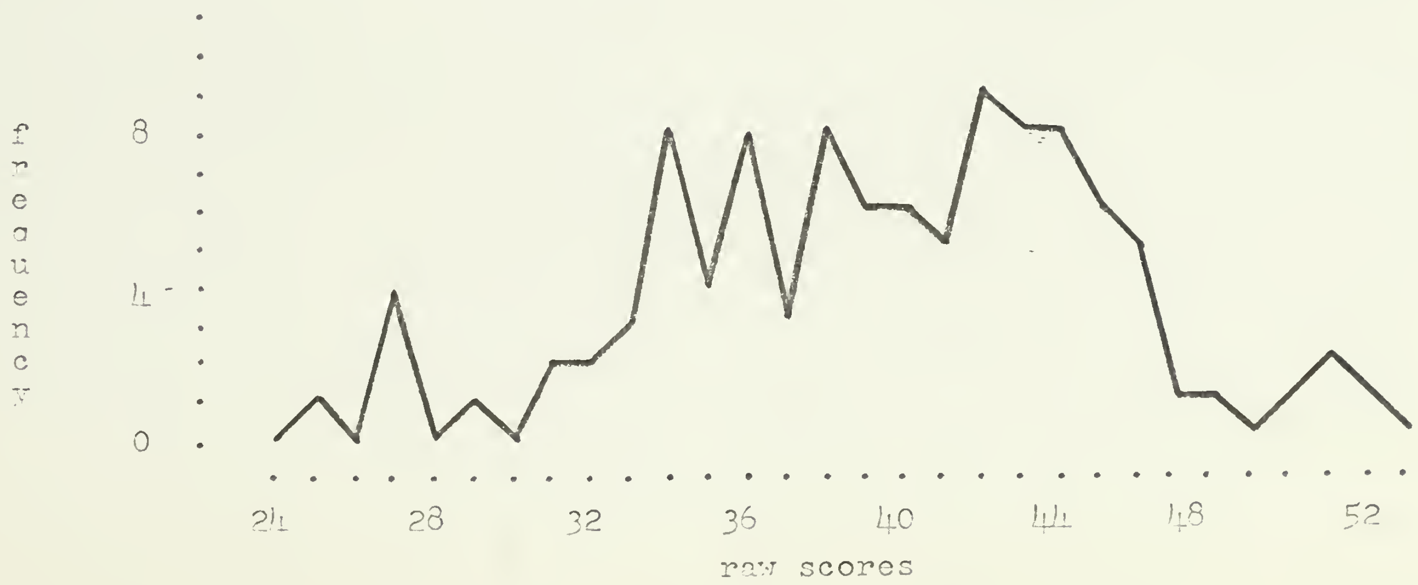
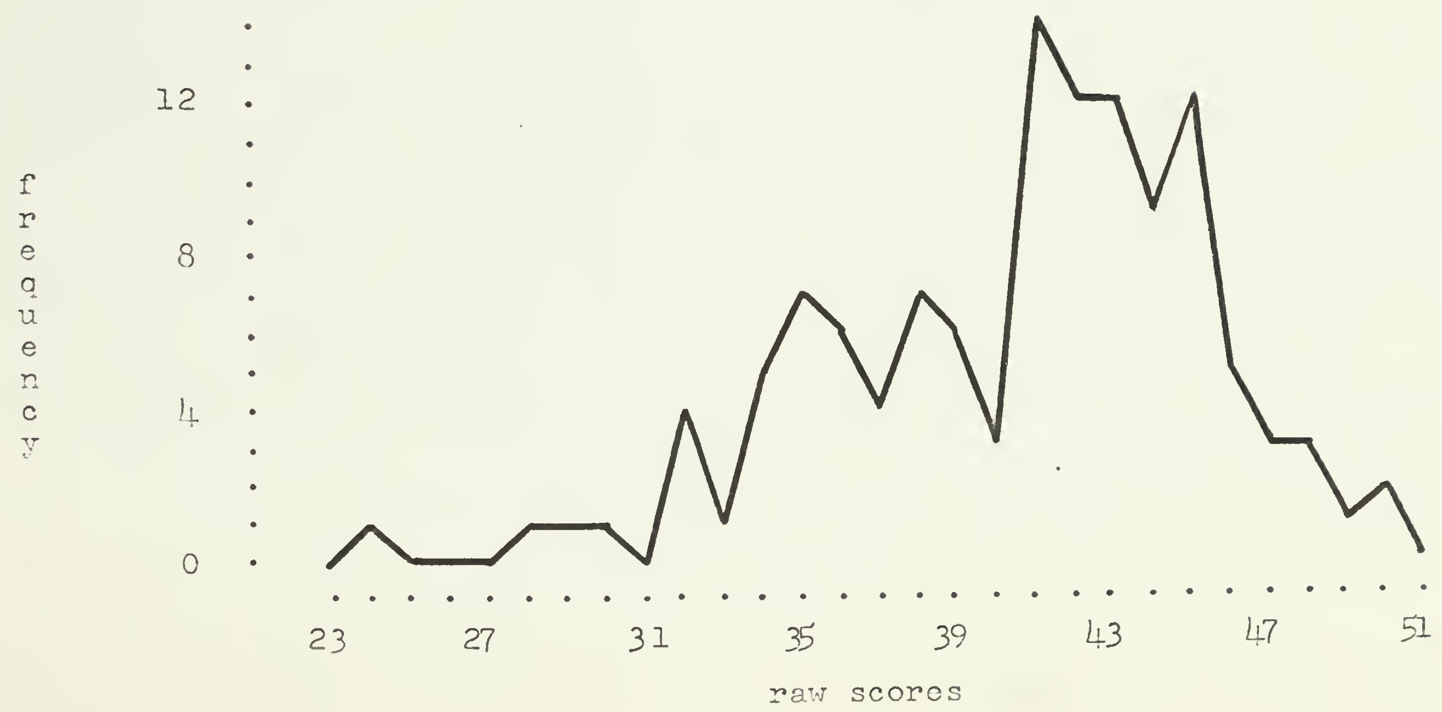
ON TEST OF QUANTITATIVE JUDGMENT



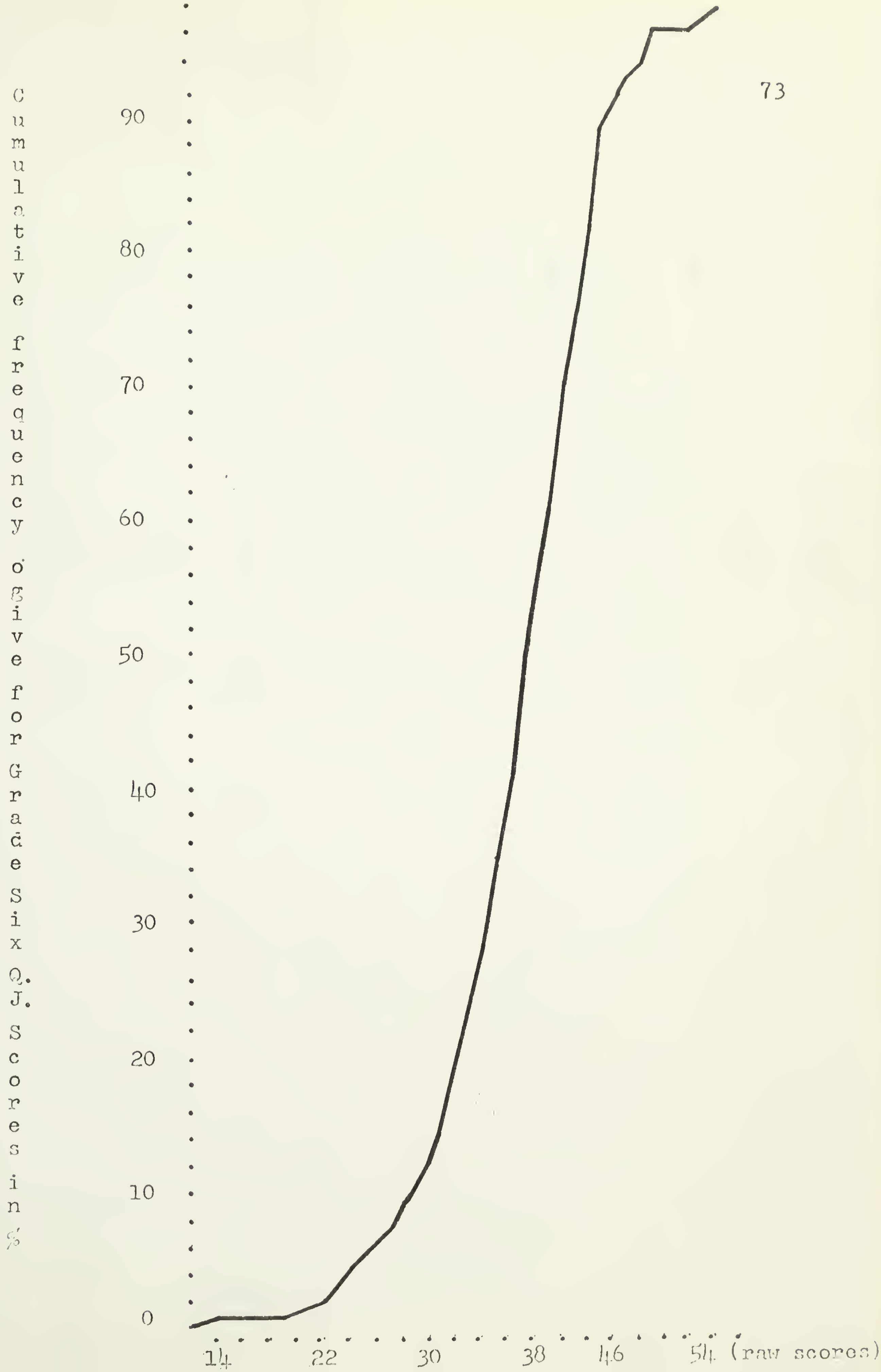
FREQUENCY POLYGON OF TOTAL EIGHTH GRADE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT

FREQUENCY POLYGON OF NINTH GRADE MALE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT

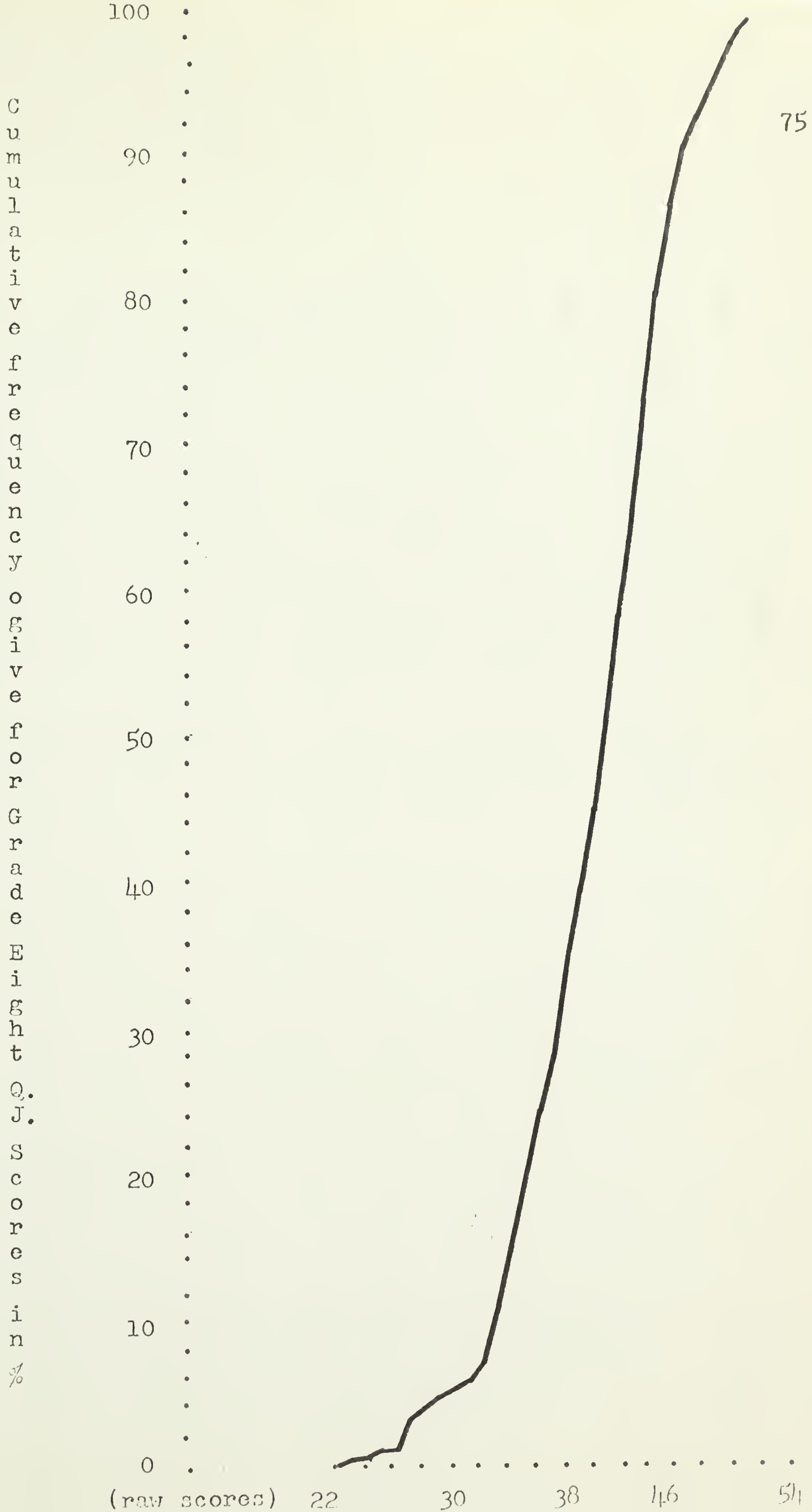
FREQUENCY POLYGON OF NINTH GRADE FEMALE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT



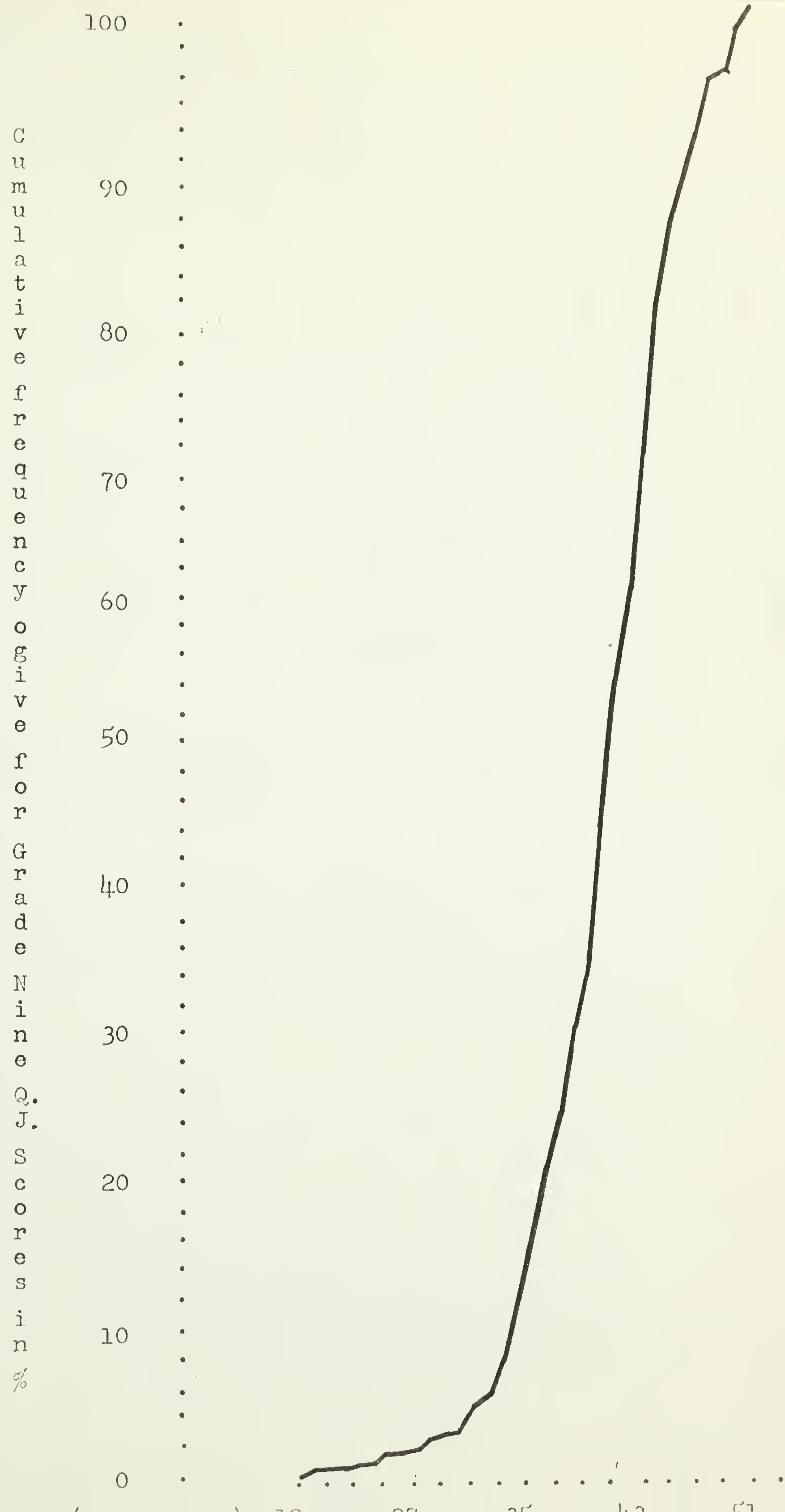
FREQUENCY POLYGON OF TOTAL NINTH GRADE RAW SCORES
ON TEST OF QUANTITATIVE JUDGMENT


Cumulative frequency ogive for Grade Six Q.J. Scores in %
90
80
70
60
50
40
30
20
10
0
14
22
30
38
46
54 (raw scores)


100
90
80
70
60
50
40
30
20
10
0
Cumulative frequency ogive for Grade Seven Q.J. Scores in %
14
22
30
38
46
54 (raw scores



Cumulative frequency ogive for Grade Eight Q. J. Scores in %
100
90
80
70
60
50
40
30
20
10
0
75
(raw scores)
22
30
38
46
54


Cumulative frequency ogive for Grade Nine Q.J. Scores in %
100
90
80
70
60
50
40
30
20
10
0

Item Analysis

Tables 8 - 19 contain the tabulations of the item difficulty coefficients (Δ), the discriminating power coefficients (r), and the proportion of success of the correct answer choices (p) by both sex and grade. These statistics were computed using Chung Teh Fan's Item Analysis Table. This table is based upon the use of the upper and lower 27 percent of the total rank ordered scores for each category.

As a result of the analysis, the writer determined that 55 items out of 60 items (92 percent) have either good or excellent high-low discrimination indices at one or more grade levels for either one or both sexes. (The following items lacked discriminating power: 22, 26, 31, 48, and 49.) Only one item was too easy for all categorizations. (Namely, item 3) None of the items were too difficult for all categorizations. (Criteria for these statistics were based on the Hall study.)

TABLE 4

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY GRADE AND SEX

Grade 6 - Boys

Item	p	r	Δ	Item	p	r	Δ
1.	.81	.33x	9.5	31.	.61	.00xx	11.9
2.	.71	.44	10.8	32.	.56	.40	12.4
3.	.90	.40	7.9*	33.	.64	.36x	11.5
4.	.76	.43	10.1	34.	.42	.03xx	13.8
5.	.86	.32x	8.6	35.	.73	.50	10.6
6.	.66	.33x	11.4	36.	.70	.54	10.9
7.	.93	.09xx	7.2*	37.	.84	.53	9.0
8.	.66	.00xx	11.3	38.	.38	.24xx	14.2
9.	.89	.60	8.1	39.	.69	.19xx	11.0
10.	.71	.44	10.8	40.	.40	.21xx	14.0
11.	.81	.46	9.5	41.	.84	.38x	9.0
12.	.80	.36x	9.7	42.	.35	.19xx	14.5
13.	.87	.48	8.6	43.	.66	.40	11.4
14.	.44	.17xx	13.6	44.	.59	.51	12.1
15.	.82	.30x	9.3	45.	.82	.57	9.3
16.	.61	.41	11.9	46.	.28	.04xx	15.3
17.	.81	.46	9.5	47.	.86	.66	8.7
18.	.66	.40	11.4	48.	.28	.04xx	15.3
19.	.78	.38x	12.3	49.	.27	.00xx	15.5
20.	.57	.38x	12.3	50.	.44	.17xx	13.6
21.	.76	.35x	10.2	51.	.63	.45	11.7
22.	.44	-.06xx	13.6	52.	.13	.00xx	17.5**
23.	.71	.23xx	10.8	53.	.88	.62	8.3
24.	.65	.49	11.5	54.	.21	-.14xx	16.3
25.	.80	.47	9.7	55.	.71	.43	10.7
26.	.36	.24xx	14.5	56.	.21	-.15xx	16.2
27.	.68	.22xx	11.2	57.	.75	.56	10.3
28.	.66	.59	11.3	58.	.10	.11xx	18.0**
29.	.88	.45	8.3	59.	.42	.42	13.8
30.	.60	.37x	12.0	60.	.17	-.05xx	16.7

* very easy
** very difficult
x questionable in discriminating power
xx poor levels of discrimination

TABLE 5

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 6 - Girls

Item	p	r	Δ	Item	p	r	Δ
1.	.81	.42	9.5	31.	.51	.10xx	12.9
2.	.57	.08xx	12.3	32.	.48	.34x	13.2
3. ***				33.	.58	.42	12.2
4.	.88	.38x	8.2	34.	.46	.07xx	13.4
5.	.86	.28xx	8.6	35.	.53	.25xx	12.7
6.	.61	.31x	11.9	36.	.63	.34x	11.7
7. ***				37.	.87	.24xx	8.4
8.	.81	.55	9.5	38.	.31	.32x	14.9
9.	.83	.50	9.5	39.	.70	.43	10.9
10.	.60	.46	12.0	40.	.50	.50	13.0
11.	.70	.19xx	10.9	41.	.87	.24xx	8.4
12.	.84	.10xx	9.0	42.	.40	.32x	14.0
13.	.88	.38x	8.2	43.	.58	.36x	12.2
14.	.39	.30x	14.2	44.	.40	.54	14.0
15.	.83	.50	9.2	45.	.75	.00xx	10.3
16.	.52	.35x	12.8	46.	.19	.04xx	16.4
17.	.90	.14xx	7.8*	47.	.70	.12xx	10.9
18.	.49	.25xx	13.1	48.	.25	.00xx	15.7
19.	.92	.52	10.8	49.	.25	.28xx	15.6
20.	.71	.40	10.8	50.	.35	.10xx	14.5
21.	.73	.45	10.5	51.	.53	.31x	12.7
22.	.44	.03xx	13.6	52.	.10	.07xx	18.0**
23.	.68	.46	11.1	53.	.74	.08xx	10.4
24.	.59	.13xx	12.1	54.	.13	.06xx	17.4**
25.	.69	.53	11.1	55.	.90	.14xx	7.8*
26.	.45	.16xx	13.5	56. ****			
27.	.75	.32x	10.3	57.	.69	.53	11.1
28.	.68	.32x	11.2	58. ****			
29.	.92	.25xx	7.4*	59.	.54	.16xx	12.6
30.	.65	.30x	11.5	60.	.21	.48	16.3

* very easy
** very difficult
*** very easy--low discrimination and values not listed
**** very difficult--low discrimination and values not listed
x questionable in discriminating power
xx poor levels of discrimination

TABLE 6

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY GRADE AND SEX

Grade 6 - Total

Item	p	r	Δ	Item	p	r	Δ
1.	.83	.41	9.2	31.	.56	.13xx	12.4
2.	.63	.29xx	11.7	32.	.51	.33x	12.9
3.	.92	.31x	7.3*	33.	.58	.40	12.2
4.	.83	.44	9.1	34.	.44	.00xx	13.6
5.	.87	.35x	8.5	35.	.61	.41	11.9
6.	.67	.36x	11.3	36.	.64	.41	11.6
7.	.88	.31x	8.2	37.	.77	.09xx	10.1
8.	.72	.18xx	10.1	38.	.38	.29xx	14.2
9.	.87	.63	8.4	39.	.68	.34x	11.2
10.	.67	.50	11.2	40.	.44	.34x	13.6
11.	.73	.33x	10.5	41.	.86	.33x	8.7
12.	.80	.28xx	9.6	42.	.56	-.12xx	12.4
13.	.87	.36x	8.6	43.	.63	.45	11.7
14.	.45	.23xx	13.5	44.	.49	.49	13.1
15.	.83	.45	9.2	45.	.82	.42	9.3
16.	.57	.40	12.3	46.	.24	.13xx	15.8
17.	.86	.38x	8.7	47.	.79	.31x	9.8
18.	.57	.27xx	12.3	48.	.30	-.05xx	15.1
19.	.84	.39x	9.0	49.	.26	.19xx	15.5
20.	.62	.33x	11.8	50.	.42	.22xx	13.8
21.	.79	.49	9.8	51.	.57	.40	12.3
22.	.40	-.07xx	14.0	52.	.12	-.04xx	17.7**
23.	.67	.31x	11.3	53.	.83	.44	9.1
24.	.63	.32x	11.7	54.	.14	.05xx	17.2**
25.	.77	.57	10.1	55.	.81	.36x	9.5
26.	.40	.24xx	14.0	56.	.11	.00xx	17.9**
27.	.72	.24xx	10.6	57.	.75	.60	10.4
28.	.68	.49	11.2	58.	.07	.03xx	18.8**
29.	.88	.31x	8.2	59.	.49	.29xx	13.1
30.	.63	.32x	11.7	60.	.18	.22xx	16.7

* very easy
** very difficult
x questionable in discriminating power
xx poor levels of discrimination

TABLE 7

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY GRADE AND SEX

Grade 7 - Boys

Item	p	r	Δ	Item	p	r	Δ
1.	.82	.52	9.4	31.	.65	.21xx	11.5
2.	.52	.45	12.8	32.	.65	.12xx	11.5
3.	.91	.28xx	7.6*	33.	.80	.38x	9.6
4.	.82	.52	9.4	34.	.52	.37x	12.8
5.	.85	.26xx	8.8	35.	.59	.31x	12.1
6.	.57	.35x	12.3	36.	.71	.35x	10.8
7.	.92	.23xx	7.3*	37.	.83	.49	9.1
8.	.78	.30x	9.9	38.	.13	.02xx	17.4**
9.	.92	.23xx	7.3*	39.	.77	.44	10.0
10.	.73	.40	10.6	40.	.47	.45	13.3
11.	.77	.44	10.0	41.	.80	.38x	9.6
12.	.76	.04xx	10.2	42.	.49	.33x	13.1
13.	.87	.42	8.6	43.	.61	.28xx	11.9
14.	.40	.31x	14.1	44.	.60	.53	12.0
15.	.87	.64	8.5	45.	.85	.46	8.8
16.	.54	.41	12.6	46.	.38	.12xx	14.2
17.	.88	.39x	8.3	47.	.81	.72	9.5
18.	.54	.34x	12.6	48.	.16	.18xx	16.9
19.	.75	.25xx	10.2	49.	.18	.00xx	16.7
20.	.69	.30x	11.1	50.	.46	.48	13.5
21.	.73	.40	10.6	51.	.54	.41	12.6
22.	.49	-.11xx	13.1	52.	.18	.12xx	16.7
23.	.84	.68	9.0	53.	.85	.66	8.8
24.	.72	.23xx	10.7	54.	.09	.55	18.4**
25.	.80	.38x	9.6	55.	.81	.72	9.5
26.	.59	.07xx	12.1	56.	.05	.16xx	19.6**
27.	.74	.09xx	10.5	57.	.85	.66	8.8
28.	.81	.55	9.5	58.	.26	.32x	15.6
29.	.88	.39x	8.3	59.	.36	.31x	14.4
30.	.72	.13xx	10.7	60.	.35	.29xx	14.6

* very easy
** very difficult
x questionable in discriminating power
xx poor levels of discrimination

TABLE 8

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 7 - Girls

Item	p	r	Δ	Item	p	r	Δ
1.	.88	.22xx	8.3	31.	.58	.21xx	12.2
2.	.49	.45	13.1	32.	.48	.00xx	13.2
3.	.94	.46	6.8*	33.	.66	.31x	11.4
4.	.87	.63	8.4	34.	.44	.07xx	13.6
5.	.95	.41	6.4*	35.	.53	.18xx	12.7
6.	.58	.15xx	12.2	36.	.47	.41	13.3
7.	.95	.41	6.4*	37.	.90	.16xx	7.9*
8.	.80	.55	9.6	38.	.27	.38x	15.4
9.	.87	.41	8.5	39.	.69	.31x	11.0
10.	.44	.42	13.6	40.	.60	.56	12.0
11.	.87	.63	8.4	41.	.82	.52	9.4
12.	.90	.33x	7.9*	42.	.52	.41	12.8
13.	.90	.33x	7.9*	43.	.69	.24xx	11.0
14.	.34	.49	14.7	44.	.56	.39x	12.4
15.	.87	.63	8.4	45.	.87	.41	8.5
16.	.42	.46	13.8	46.	.22	.37x	16.0
17.	.69	.31x	11.0	47.	.90	.16xx	7.9*
18.	.58	.35x	12.2	48.	.28	-.21xx	15.3
19.	.81	.42	9.5	49.	.31	.00xx	15.0
20.	.75	.31x	10.3	50.	.34	.07xx	14.7
21.	.72	.45	10.7	51.	.68	.43	11.1
22.	.51	.28xx	12.9	52.	.06	.00xx	19.2**
23.	.66	.31x	11.4	53.	.83	.70	9.2
24.	.74	.04xx	10.5	54.	.16	-.12xx	16.9
25.	.72	.57	10.7	55.	.90	.57	7.9
26.	.42	.11xx	13.8	56.	.11	-.24xx	17.9**
27.	.82	.00xx	9.3	57.	.71	.28xx	10.8
28.	.80	.55	9.6	58.	.11	.24xx	17.9**
29.	.81	.16xx	9.5	59.	.49	.03xx	13.1
30.	.70	.40	10.9	60.	.14	.51	17.2**

* very easy
** very difficult
x questionable in discriminating power
xx poor levels of discrimination

TABLE 9

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 7 - Total

Item	p	r	Δ	Item	p	r	Δ
1.	.84	.29xx	9.0	31.	.53	.08xx	12.7
2.	.51	.40	12.9	32.	.59	.05xx	12.1
3.	.92	.40	7.3*	33.	.71	.33x	10.8
4.	.79	.40	9.7	34.	.45	.10xx	13.5
5.	.91	.30x	7.7*	35.	.53	.22xx	12.7
6.	.57	.20xx	12.3	36.	.61	.42	11.9
7.	.94	.32x	6.7*	37.	.87	.31x	8.5
8.	.79	.38x	9.8	38.	.36	.50	14.4
9.	.90	.33x	7.9*	39.	.71	.46	10.8
10.	.64	.57	11.6	40.	.54	.43	12.6
11.	.83	.49	9.1	41.	.80	.47	9.7
12.	.83	.14xx	9.2	42.	.48	.39x	13.2
13.	.88	.38x	8.2	43.	.67	.31x	11.3
14.	.40	.39x	14.0	44.	.59	.48	12.0
15.	.87	.63	8.4	45.	.88	.52	8.3
16.	.44	.41	13.6	46.	.32	.25xx	14.9
17.	.80	.36x	9.7	47.	.83	.29xx	9.2
18.	.55	.31x	12.5	48.	.21	-.07xx	16.2
19.	.78	.30x	10.9	49.	.22	.04xx	16.0
20.	.70	.30x	10.9	50.	.38	.30x	14.3
21.	.73	.43	10.5	51.	.63	.42	11.6
22.	.48	.11xx	13.2	52.	.11	.15xx	17.8**
23.	.78	.51	10.0	53.	.86	.67	8.9
24.	.74	.15xx	10.4	54.	.13	.13xx	17.4**
25.	.76	.47	10.2	55.	.86	.65	8.7
26.	.52	.10xx	12.8	56.	.11	-.04xx	17.9**
27.	.78	.00xx	9.9	57.	.79	.49	9.8
28.	.78	.51	10.0	58.	.16	.20xx	17.0**
29.	.83	.31x	9.1	59.	.48	.17xx	13.2
30.	.66	.25xx	11.4	60.	.22	.36x	16.1

* very easy
** very difficult
x questionable in discriminating power
xx poor levels of discrimination

TABLE 10

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 8 - Boys

Item	p	r	Δ	Item	p	r	Δ
1.	.85	.45	8.8	31.	.78	.00xx	9.9
2.	.63	.34x	11.7	32.	.57	.22xx	12.3
3.	.95	.26xx	6.3*	33.	.82	.71	9.4
4.	.81	.37x	9.5	34.	.47	.11xx	13.3
5.	.95	.41	6.4*	35.	.74	.40	10.4
6.	.57	.15xx	12.3	36.	.65	.30x	11.5
7.	.91	.07xx	7.8*	37.	.91	.37x	7.7*
8.	.87	.41	8.5	38.	.66	.54	11.3
9.	.95	.41	6.4*	39.	.76	.46	10.1
10.	.81	.72	9.5	40.	.59	.26xx	12.1
11.	.94	.47	6.9*	41.	.87	.20xx	8.4
12.	.93	.21xx	7.2*	42.	.57	.37x	12.3
13.	.93	.21xx	7.2*	43.	.83	.33x	9.3
14.	.63	.34x	11.7	44.	.76	.38x	12.3
15. ***				45.	.95	.41	6.4*
16.	.63	.34x	11.7	46.	.30	.21xx	15.1
17.	.90	.57	7.9*	47.	.90	.33x	7.9
18.	.40	.03xx	14.0	48.	.19	.16xx	16.5
19.	.89	.00xx	11.3	49.	.28	.08xx	15.3
20.	.66	.54	11.3	50.	.53	.22xx	12.7
21.	.79	.31x	9.8	51.	.64	.57	11.6
22.	.64	.00xx	11.6	52.	.13	.48	17.4**
23.	.74	.40	10.4	53.	.88	.38x	8.2
24.	.85	.67	8.9	54.	.18	.39x	16.6
25.	.94	.47	6.9*	55.	.90	.57	7.9*
26.	.47	.25xx	13.3	56.	.17	.24xx	16.8
27.	.81	.27xx	9.5	57.	.80	.56	9.7
28.	.87	.63	8.5	58.	.22	.47	16.0
29.	.91	.07xx	7.8*	59.	.49	.29xx	13.1
30.	.66	.12xx	11.4	60.	.35	.49	14.5

* very easy

** very difficult

*** very easy--low discrimination and values not listed

x questionable in discriminating power

xx poor levels of discrimination

TABLE 11

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 8 - Girls

Item	p	r	Δ	Item	p	r	Δ
1.	.85	.45	8.8	31.	.59	.03xx	12.1
2.	.55	.47	12.4	32.	.40	-.03xx	14.0
3. ***				33.	.71	.17xx	10.8
4.	.87	.41	8.5	34.	.61	-.07xx	11.9
5. ***				35.	.66	.62	11.3
6.	.49	.35x	13.1	36.	.63	.27xx	11.7
7. ***				37.	.92	.51	7.2*
8.	.75	.27xx	10.3	38.	.39	.30x	14.2
9.	.89	.61	8.2	39.	.57	.37x	12.3
10.	.69	.40	11.1	40.	.57	.30x	12.3
11.	.81	.54	9.5	41.	.89	.15xx	8.2
12.	.95	.41	6.4*	42.	.46	.36x	13.4
13.	.95	.41	6.4*	43.	.79	.40	7.9*
14.	.42	.36x	13.8	44.	.75	.49	10.3
15.	.87	.41	8.5	45.	.90	.33x	7.9*
16.	.46	.29xx	13.4	46.	.33	.37x	14.8
17.	.79	.40	9.7	47.	.85	.45	8.8
18.	.40	-.03xx	14.0	48.	.19	-.06xx	16.5
19.	.87	.08	8.5	49.	.28	.08xx	15.3
20.	.63	.34x	11.7	50.	.49	.15xx	13.1
21.	.72	.54	10.7	51.	.69	.71	11.1
22.	.54	.07xx	12.6	52.	.11	.41	18.0**
23.	.80	.56	9.7	53.	.87	.63	8.5
24.	.65	.46	11.5	54.	.19	-.06xx	16.5
25.	.86	.25xx	8.5	55.	.91	.07xx	7.8*
26.	.64	.24xx	11.5	56.	.11	.41	18.0**
27.	.79	.10xx	9.8	57.	.84	.48	9.1
28.	.75	.49	10.5	58.	.11	.41	18.0**
29.	.94	.14xx	6.7*	59.	.79	.40	9.7
30.	.76	.46	10.1	60.	.24	.41	15.8

* very easy
** very difficult
*** very easy--low discrimination and values not listed
x questionable in discriminating power
xx poor levels of discrimination

TABLE 12

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 8 - Total

Item	p	r	Δ	Item	p	r	Δ
1.	.87	.54	8.4	31.	.70	.07xx	10.9
2.	.58	.42	12.2	32.	.49	.11xx	13.1
3.	.95	.26xx	6.3*	33.	.77	.45	10.1
4.	.83	.33x	9.3	34.	.59	-.03xx	12.1
5.	.95	.26xx	6.3*	35.	.68	.51	11.1
6.	.57	.26xx	12.3	36.	.67	.30x	11.2
7.	.94	.16xx	6.9*	37.	.90	.33x	7.9*
8.	.82	.42	9.3	38.	.59	.60	12.1
9. ***				39.	.66	.39x	11.3
10.	.80	.73	9.6	40.	.59	.30x	12.1
11.	.89	.59	8.0	41.	.91	.28xx	7.6*
12.	.95	.28xx	6.4*	42.	.51	.41	12.9
13.	.95	.28xx	6.4*	43	.81	.37x	9.5
14.	.49	.43	13.1	44.	.71	.46	10.8
15.	.89	.25xx	8.2	45.	.93	.39x	7.2*
16.	.57	.48	12.3	46.	.30	.21xx	15.1
17.	.86	.43	8.6	47.	.88	.52	8.3
18.	.41	.05xx	13.9	48.	.24	.09xx	15.8
19.	.89	.40	11.1	49.	.24	.09xx	15.8
20.	.69	.40	11.1	50.	.47	.21xx	13.3
21.	.74	.36x	10.5	51.	.65	.52	11.5
22.	.53	.10xx	12.7	52.	.13	.21xx	17.5**
23.	.79	.48	9.7	53.	.87	.54	8.4
24.	.75	.56	10.3	54.	.22	.17xx	16.1
25.	.91	.45	7.7*	55.	.91	.43	7.5*
26.	.53	.14xx	12.7	56.	.12	.18xx	17.7**
27.	.80	.23xx	9.7	57.	.82	.52	9.4
28.	.82	.63	9.3	58.	.15	.52	17.1**
29.	.91	.07xx	7.8*	59.	.59	.18xx	12.1
30.	.70	.19xx	10.9	60.	.28	.53	15.3

* very easy
** very difficult
*** very easy--low discrimination and values not listed
x questionable in discriminating power
xx poor levels of discrimination

TABLE 13

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 9 - Boys

Item	p	r	Δ	Item	p	r	Δ
1. ***				31.	.78	.05xx	9.9
2.	.66	.33x	11.4	32.	.62	.24xx	11.8
3.	.92	.19xx	7.5*	33.	.87	.40	8.4
4.	.82	.19xx	9.3	34.	.52	.35x	12.8
5.	.94	-.14xx	6.7*	35.	.75	.24xx	10.3
6.	.63	.22xx	11.6	36.	.77	.20xx	10.1
7.	.91	.15xx	7.6*	37.	.94	.10xx	6.9*
8.	.80	.19xx	9.7	38.	.41	.24xx	13.9
9. ***				39.	.71	.13xx	10.8
10.	.83	.50	9.2	40.	.47	.56	13.3
11.	.85	.28xx	8.9	41.	.90	.09xx	7.9*
12.	.90	-.09xx	7.9*	42.	.62	.48	11.8
13.	.92	.23xx	7.3*	43.	.86	.07xx	8.7
14.	.62	.67	11.8	44.	.73	.51	10.5
15.	.92	.19xx	7.5*	45.	.89	.35x	9.1
16.	.65	.25xx	11.5	46.	.30	.36x	15.2
17.	.85	.28xx	8.9	47.	.86	.22	8.6
18.	.22	.21xx	16.1	48.	.26	.19xx	15.5
19.	.86	.07xx	8.7	49.	.32	.05xx	14.9
20.	.74	.24xx	10.3	50.	.67	.30x	11.2
21.	.83	.31x	9.1	51.	.63	.28xx	11.6
22.	.40	.13xx	14.0	52.	.09	-.10xx	18.4**
23.	.89	.35x	8.1	53.	.88	.16xx	8.2
24.	.94	-.14xx	6.7*	54.	.13	.24xx	17.6**
25.	.85	.47	8.9	55.	.86	.22xx	8.6
26.	.38	-.09xx	14.2	56.	.15	.00xx	17.1**
27.	.77	.20xx	10.1	57.	.90	.09xx	7.9*
28.	.79	.27xx	9.7	58.	.16	.68	16.9
29.	.94	.14xx	6.7*	59.	.55	.04xx	12.5
30.	.87	.40	8.4	60.	.35	.66	14.6

* very easy
** very difficult
*** very easy--low discrimination and values not listed
x questionable in discriminating power
xx poor levels of discrimination

TABLE 14

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 9 - Girls

Item	p	r	Δ	Item	p	r	Δ
1.	.91	.30x	7.7*	31.	.65	.13xx	11.4
2.	.53	.25xx	12.7	32.	.46	.07xx	13.4
3.	.92	.25xx	7.4*	33.	.68	.07xx	11.1
4.	.81	.18xx	9.4	34.	.63	.27xx	11.7
5. ***				35.	.71	.40	10.8
6.	.73	.45	10.5	36.	.51	.28xx	12.9
7.	.94	.45	6.7*	37. ***			
8.	.79	.44	9.7	38.	.40	.26xx	14.0
9.	.93	.49	7.0*	39.	.69	.29xx	11.0
10.	.74	.65	10.4	40.	.59	.60	12.1
11.	.89	.19xx	8.1	41.	.87	.41	8.5
12.	.94	.45	6.7*	42.	.51	.41	12.9
13.	.94	.45	6.7*	43.	.75	.32x	10.3
14.	.51	.16xx	12.9	44.	.51	.40	12.9
15.	.80	.31x	9.6	45.	.89	.61	8.2
16.	.61	.43	11.8	46.	.16	.27xx	16.9
17.	.89	.19xx	8.1	47.	.87	.41	8.5
18.	.51	.16xx	12.9	48.	.16	.05xx	16.9
19. ***				49.	.27	.23xx	15.4
20.	.70	.34x	10.9	50.	.53	.37x	12.7
21.	.72	.48	10.7	51.	.67	.25xx	11.2
22.	.37	.19xx	14.3	52.	.14	.34x	17.3**
23.	.86	.28xx	8.6	53.	.92	.25xx	7.4*
24.	.79	.35x	9.8	54.	.17	.40	16.8
25.	.82	.38x	9.3	55.	.84	.20xx	9.0
26.	.58	.09xx	12.2	56.	.10	.07xx	18.0**
27.	.90	.14xx	7.8*	57.	.75	.43	10.4
28.	.79	.35x	9.8	58.	.15	.00xx	17.1**
29.	.90	.00xx	7.9*	59.	.64	.10xx	11.6
30.	.66	.28xx	11.4	60.	.33	.36x	14.7

* very easy
** very difficult
*** very easy--low discrimination and values not listed
x questionable in discriminating power
xx poor levels of discrimination

TABLE 15

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 9 - Total

Item	p	r	Δ	Item	p	r	Δ
1.	.93	.09xx	7.2*	31.	.71	.20xx	10.8
2.	.60	.30x	12.0	32.	.50	.10xx	13.0
3.	.92	.23xx	7.3*	33.	.75	.25xx	10.4
4.	.82	.14xx	9.4	34.	.56	.28xx	12.4
5.	.94	.00xx	6.8*	35.	.70	.38x	10.9
6.	.69	.33x	11.0	36.	.64	.26xx	11.5
7.	.92	.23xx	7.3*	37.	.95	.11xx	6.6*
8.	.78	.18xx	9.9	38.	.39	.31x	14.1
9.	.92	.26xx	7.5*	39.	.70	.28xx	11.0
10.	.75	.52	10.3	40.	.56	.55	12.3
11.	.87	.16xx	8.5	41.	.86	.32x	8.6
12.	.91	.10xx	7.6*	42.	.57	.39x	12.2
13.	.95	.28xx	6.4*	43.	.79	.27xx	9.7
14.	.67	.11xx	11.2	44.	.60	.50	12.0
15.	.86	.29xx	8.7	45.	.90	.48	8.0
16.	.59	.43	12.1	46.	.23	.35x	16.0
17.	.85	.21xx	8.8	47.	.86	.32x	8.6
18.	.41	.07xx	13.9	48.	.21	.18xx	16.2
19.	.92	.05xx	7.4*	49.	.29	.10xx	15.2
20.	.78	.34x	10.0	50	.53	.36x	12.7
21.	.79	.44	9.7	51.	.66	.25xx	11.4
22.	.41	.08xx	13.9	52.	.21	-.17xx	16.3
23.	.85	.30x	8.8	53.	.90	.34x	8.0
24.	.85	.14xx	8.8	54.	.15	.36x	17.1**
25.	.85	.30x	8.8	55.	.86	.29xx	8.7
26.	.49	-.05xx	13.1	56.	.13	.08xx	17.5**
27.	.85	.04xx	8.9	57.	.81	.33x	9.5
28.	.82	.31x	9.4	58.	.18	.28xx	16.6
29.	.91	.12xx	7.7*	59.	.62	.20xx	11.8
30.	.80	.35x	9.6	60.	.34	.44	14.7

* very easy
** very difficult
*** very easy--low discrimination and values not listed
x questionable in discriminating power
xx poor levels of discrimination

Hypothesis Testing

The testing of the hypotheses invoked the use of the BMD 03R--Multiple Linear Regression Analysis[3] and The Least-Squares and Maximum Likelihood General Purpose Program[4] which was an analysis of covariance model combining a two-way ANOVA (with interaction) with linear regression.

Results of the Multiple Linear Regression Analysis. A multiple linear regression analysis was performed on the scores of the California Test of Mental Maturity (CTMM), The Iowa Tests of Basic Skills (ITBS)(vocabulary, reading, arithmetic concepts and arithmetic reasoning), previous mathematics grades, and the Test of Quantitative Judgment. The scores on the Test of Quantitative Judgment were used as the dependent variable in testing the first question which was considering whether or not the Test of Quantitative Judgment was measuring something unique from the other variables. These analyses demonstrated that the Test of Quantitative Judgment was measuring an attribute unique from the other variables used in the study. The scores on the Test of Quantitative Judgment were significantly related to IQ test and Arithmetic Concepts Test scores. (Intercorrelation matrices are displayed in

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[3]BMD 03R, Multiple Linear Regression Analysis (version of August 13, 1964), Health Sciences Computing Facility, UCLA.

[4]Walther R. Harvey, Least-Squares and Maximum Likelihood General Pupose Computer Program (Ohio State University, 1968).

Tables 30-34.

A separate regression analysis was performed by grade, sex, and on the composite of the total sample. In all cases, except the ninth grade males, an F-value significant at the 0.01 level was found. The F-value for the ninth grade males was significant at the 0.05 level. However, in all cases, the multiple correlation coefficient yielded an important amount of unexplained variance (ie., attributable to the Test of Quantitative Judgment).

Table 16 lists the percentages of explained and unexplained variance resulting from the multiple linear regression analyses with respect to the Test of Quantitative Judgment. (The unexplained variance was greater than 47 percent in all cases.)

Tables 17-29 report the results of the separate multiple linear regression analyses for grade, sex, and the total population.

Note that the amounts of variance explained by each of these analyses decreases as the grade level increases. Within this grade span, the means on the Test of Quantitative Judgment increase by grade as the standard deviations, in general, decrease between grades. (Except in the aforementioned situation between the sixth and seventh grade levels discussed on page 52)

The results of the multiple linear regression analyses seem to support the following conclusions: (1) There is no real indication from the distribution of the scores of the various standardized tests used in the study to suspect that the sample was not approximately normally distributed; (2) These regression analyses indicate that the standardized tests used in the study serve as better predictors of student achievement at the sixth grade level than at the ninth grade level; (3) It appears that as a student progresses through the higher grades, the better predictor of achievement is estimated by actual student performance on aspects of the curriculum rather than the results of standardized tests; and (4) The results of the multiple linear regression analyses (observable differences in the means of QJ and the decreases in standard deviation on QJ at each grade level) apparently indicate that quantitative judgment can be learned or acquired (at least in part).

Tables 30-34 display the intercorrelation matrices for the California Test of Mental Maturity (IQ), the Iowa Tests of Basic Skills (ITBS), previous mathematics grades, and the Test of Quantitative Judgment(QJ).

TABLE 16

PERCENTAGE OF VARIANCE EXPLAINED AND UNEXPLAINED RESULTING FROM MULTIPLE LINEAR REGRESSION ANALYSIS WITH RESPECT TO QJ

	% Variance Explained	% Variance Unexplained
Grade Six		
Male	43.7	56.3
Female	52.7	47.3
Total	44.9	55.1
Grade Seven		
Male	42.5	57.5
Female	43.9	56.1
Total	42.1	57.9
Grade Eight		
Male	37.3	62.7
Female	37.9	62.1
Total	36.9	63.1
Grade Nine		
Male	29.4	70.6
Female	32.4	67.6
Total	29.5	70.5
OVERALL	31.1	68.9

TABLE 17

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE SIX BOYS

	MEAN	S.D.
IQ	110.30	18.14
VC	61.82	22.68
RD	55.59	24.81
AC	59.83	25.01
AR	52.16	27.40
PG	30.53	10.19
QJ	37.32	6.26

ANALYSIS OF VARIANCE FOR MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	2244.54447	374.09074	16.1911 *
Deviation about Regression	125	2888.09189	23.10474	
Total	131	5132.63636		

Multiple Correlation Coefficient	0.6613
Variance of Estimate	23.10474
Standard Error of Estimate	4.80674

Code:
- IQ - California Test of Mental Maturity
- VC - Iowa Vocabulary
- RD - Iowa Reading Comprehension
- AC - Iowa Arithmetic Concepts
- AR - Iowa Arithmetic Reasoning
- PG - Previous Mathematics Grades
- QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 18

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE SIX GIRLS

	MEAN	S.D.
IQ	115.09	14.65
VC	63.06	22.52
RD	62.74	23.51
AC	64.18	22.44
AR	66.03	27.79
PG	34.06	9.78
QJ	36.47	5.55

ANALYSIS OF VARIANCE FOR MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	1887.04566	314.50761	20.5426*
Deviation about Regression	110	1684.09964	15.31000	
Total	116	3571.14530		

Multiple Correlation Coefficient	0.7269
Variance of Estimate	15.31000
Standard Error of Estimate	3.91280

Code:
- IQ - California Test of Mental Maturity
- VC - Iowa Vocabulary
- RD - Iowa Reading Comprehension
- AC - Iowa Arithmetic Concepts
- AR - Iowa Arithmetic Reasoning
- PG - Previous Mathematics Grades
- QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 19

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE SIX

	MEAN	S.D.
IQ	112.55	16.73
VC	62.40	22.57
RD	58.95	24.42
AC	61.88	23.89
PG	32.19	10.14
QJ	36.92	5.94
AR	58.95	28.39

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	3936.28975	656.04829	32.99 *
Deviation about Regression	242	4812.10383	19.88473	
Total	248	8748.39358		

Multiple Correlation Coefficient	0.6708
Variance of Estimate	19.88473
Standard Error of Estimate	4.45932

Code:
- IQ - California Test of Mental Maturity
- VC - Iowa Vocabulary
- RD - Iowa Reading Comprehension
- AC - Iowa Arithmetic Concepts
- AR - Iowa Arithmetic Reasoning
- PG - Previous Mathematics Grades
- QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 20

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE SEVEN BOYS

	MEAN	S.D.
IQ	109.62	15.45
VC	58.92	25.16
RD	58.50	27.11
AC	67.24	22.46
AR	65.08	27.70
PG	32.28	9.23
QJ	38.74	6.94

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	2046.99543	341.16590	11.6013*
Deviation about Regression	94	2764.31150	29.40757	
Total	100	4811.30693		

Multiple Correlation Coefficient	0.6523
Variance of Estimate	29.40757
Standard Error of Estimate	5.42287

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 21

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE SEVEN GIRLS

	MEAN	S.D.
IQ	108.22	14.04
VC	61.18	24.44
RD	59.10	25.63
AC	65.63	22.96
AR	69.20	25.73
PG	34.72	10.36
QJ	37.72	5.91

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	1658.85039	276.47506	13.3722 *
Deviation about Regression	102	2108.89273	20.67542	
Total	108	3767.74312		

Multiple Correlation Coefficient	0.6635
Variance of Estimate	20.67542
Standard Error of Estimate	4.54702

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades

* significant at 0.01 level

TABLE 22

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE SEVEN

	MEAN	S.D.
IQ	108.90	14.71
VC	60.10	24.75
RD	58.81	26.29
AC	66.40	22.68
AR	67.22	26.71
PG	33.55	9.89
QJ	38.21	6.43

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	3637.80530	606.30088	24.6377 *
Deviation about Regression	203	4995.55185	24.60863	
Total	209			

Multiple Correlation Coefficient	0.6491
Variance of Estimate	24.60863
Standard Error of Estimate	4.96071

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 23

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE EIGHT BOYS

	MEAN	S.D.
IQ	110.02	13.54
VC	57.92	21.72
RD	55.14	23.23
AC	69.06	22.23
AR	60.73	26.25
PG	31.54	9.11
QJ	41.91	5.42

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	1130.93709	188.48952	9.6570 *
Deviation about Regression	97	1893.28406	19.51839	
Total	103	3024.22115		

Multiple Correlation Coefficient	0.6115
Variance of Estimate	19.51839
Standard Error of Estimate	4.41796

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 24

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE EIGHT GIRLS

	MEAN	S.D.
IQ	108.24	12.46
VC	57.54	22.37
RD	58.28	21.64
AC	66.03	20.70
AR	60.01	26.74
PG	34.86	8.43
QJ	39.29	5.62

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	1233.08782	205.51464	9.8949 *
Deviation about Regression	97	2014.67180	20.76981	
Total	103	3247.75962		

Multiple Correlation Coefficient	0.6162
Variance of Estimate	20.76981
Standard Error of Estimate	4.55739

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 25

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE EIGHT

	MEAN	S.D.
IQ	109.13	13.01
VC	57.73	22.00
RD	56.71	22.45
AC	67.54	21.48
AR	60.37	26.44
PG	33.20	8.91
QJ	40.61	5.66

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	2429.46726	404.91121	19.3862 *
Deviation about Regression	201	4198.20582	20.88660	
Total	207	6627.67308		

Multiple Correlation Coefficient	0.6054
Variance of Estimate	20.88660
Standard Error of Estimate	4.57019

Code:
- IQ - California Test of Mental Maturity
- VC - Iowa Vocabulary
- RD - Iowa Reading Comprehension
- AC - Iowa Arithmetic Concepts
- AR - Iowa Arithmetic Reasoning
- PG - Previous Mathematics Grades
- QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 26

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE NINE BOYS

	MEAN	S.D.
IQ	109.24	12.11
VC	66.74	23.11
RD	58.38	22.97
AC	59.63	22.51
AR	59.53	22.88
PG	29.54	7.91
QJ	42.67	4.98

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	652.81629	108.80272	5.6517 **
Deviation about Regression	90	1732.62700	19.25141	
Total	96	2385.44330		

Multiple Correlation Coefficient	0.5231
Variance of Estimate	19.25141
Standard Error of Estimate	4.38764

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

** significant at 0.05 level

TABLE 27

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE NINE GIRLS

	MEAN	S.D.
IQ	106.01	12.25
VC	59.63	22.35
RD	59.70	22.05
AC	53.22	21.21
AR	56.07	23.71
PG	31.17	8.32
QJ	40.58	4.84

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	903.83724	150.63954	9.0480 *
Deviation about Regression	113	1881.32943	16.64893	
Total	119	2785.16667		

Multiple Correlation Coefficient	0.5697
Variance of Estimate	16.64893
Standard Error of Estimate	4.08031

Code:
- IQ - California Test of Mental Maturity
- VC - Iowa Vocabulary
- RD - Iowa Reading Comprehension
- AC - Iowa Arithmetic Concepts
- AR - Iowa Arithmetic Reasoning
- PG - Previous Mathematics Grades
- QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 28

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR GRADE NINE

	MEAN	S.D.
IQ	107.45	12.26
VC	62.81	22.92
RD	59.11	22.43
AC	56.08	21.98
AR	57.61	23.35
PG	30.44	8.16
QJ	41.52	5.00

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	1595.31316	265.88553	14.6594 *
Deviation about Regression	210	3808.88039	18.13753	
Total	216	5404.19355		

Multiple Correlation Coefficient	0.5433
Variance of Estimate	18.13753
Standard Error of Estimate	4.25882

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading
AC - Arithmetic Concepts
AR - Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 29

RESULTS OF MULTIPLE LINEAR REGRESSION ON CTMM, ITBS, PREVIOUS GRADES, AND QJ FOR TOTAL SAMPLE

	MEAN	S.D.
IQ	109.63	14.50
VC	60.86	23.11
RD	58.43	23.94
AC	62.86	22.98
AR	60.84	26.57
PG	32.32	9.40
QJ	39.22	6.07

ANALYSIS OF VARIANCE FOR THE MULTIPLE LINEAR REGRESSION ON QJ

Source of Variation	d.f.	Sum of Squares	Mean Squares	F
Due to Regression	6	10113.00218	1685.50036	66.0667 *
Deviation about Regression	877	22374.09624	25.51208	
Total	883	32487.09842		

Multiple Correlation Coefficient	0.5579
Variance of Estimate	25.51208
Standard Error of Estimate	5.05095

Code: IQ - California Test of Mental Maturity
VC - Iowa Vocabulary
RD - Iowa Reading
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
QJ - Test of Quantitative Judgment

* significant at 0.01 level

TABLE 30

MATRIX TO SHOW INTERCORRELATIONS BETWEEN THE CTMM, ITBS, PREV. GRADES, AND QJ. FOR GRADE SIX

(MALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.57	1.00					
AR	.44	.67	1.00				
AC	.58	.74	.65	1.00			
RD	.56	.65	.62	.69	1.00		
VC	.59	.62	.53	.68	.75	1.00	
IQ	.40	.52	.44	.45	.46	.41	1.00

(FEMALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.68	1.00					
AR	.61	.77	1.00				
AC	.67	.82	.79	1.00			
RD	.60	.68	.67	.72	1.00		
VC	.60	.66	.61	.66	.81	1.00	
IQ	.55	.71	.56	.63	.59	.50	1.00

(TOTAL)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.59	1.00					
AR	.48	.73	1.00				
AC	.60	.77	.71	1.00			
RD	.56	.67	.66	.70	1.00		
VC	.59	.63	.56	.67	.77	1.00	
IQ	.41	.61	.50	.52	.52	.45	1.00

Code:
QJ - Test of Quantitative Judgment
PG - Previous Mathematics Grades
AR - Iowa Arithmetic Reasoning
AC - Iowa Arithmetic Concepts
RD - Iowa Reading Comprehension
VC - Iowa Vocabulary
IQ - California Test of Mental Maturity (CTMM)

TABLE 31

MATRIX TO SHOW INTERCORRELATIONS BETWEEN THE CTMM, ITBS, PREV. GRADES, AND QJ. FOR GRADE SEVEN

(MALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.60	1.00					
AR	.47	.67	1.00				
AC	.56	.74	.72	1.00			
RD	.56	.65	.64	.76	1.00		
VC	.57	.67	.56	.66	.81	1.00	
IQ	.57	.71	.55	.73	.69	.71	1.00

(FEMALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.54	1.00					
AR	.51	.67	1.00				
AC	.58	.76	.82	1.00			
RD	.53	.62	.57	.66	1.00		
VC	.55	.65	.59	.69	.81	1.00	
IQ	.61	.60	.58	.65	.72	.69	1.00

(TOTAL)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.53	1.00					
AR	.48	.67	1.00				
AC	.56	.74	.77	1.00			
RD	.55	.63	.60	.71	1.00		
VC	.56	.66	.58	.67	.84	1.00	
IQ	.59	.64	.56	.69	.70	.69	1.00

Code:
- QJ - Test of Quantitative Judgment
- PG - Previous Mathematics Grades
- AR - Iowa Arithmetic Reasoning
- AC - Iowa Arithmetic Concepts
- RD - Iowa Reading Comprehension
- VC - Iowa Vocabulary
- IQ - California Test of Mental Maturity (CTMM)

TABLE 32

MATRIX TO SHOW INTERCORRELATIONS BETWEEN THE CTMM, ITBS, PREV. GRADES, AND QJ. FOR GRADE EIGHT

(MALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.41	1.00					
AR	.44	.64	1.00				
AC	.58	.60	.73	1.00			
RD	.48	.44	.58	.67	1.00		
VC	.43	.30	.43	.52	.78	1.00	
IQ	.47	.45	.63	.63	.71	.63	1.00

(FEMALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.38	1.00					
AR	.50	.50	1.00				
AC	.56	.60	.68	1.00			
RD	.52	.59	.69	.74	1.00		
VC	.51	.53	.62	.70	.82	1.00	
IQ	.51	.57	.59	.59	.64	.65	1.00

(TOTAL)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.33	1.00					
AR	.46	.55	1.00				
AC	.57	.58	.70	1.00			
RD	.47	.51	.63	.70	1.00		
VC	.46	.40	.53	.60	.79	1.00	
IQ	.49	.48	.61	.61	.67	.64	1.00

Code: QJ - Test of Quantitative Judgment
PG - Previous Mathematics Grades
AR - Iowa Arithmetic Reasoning
AC - Iowa Arithmetic Concepts
RD - Iowa Reading Comprehension
VC - Iowa Vocabulary
IQ - California Test of Mental Maturity (CTMM)

TABLE 33

MATRIX TO SHOW INTERCORRELATIONS BETWEEN THE CTMM, ITBS, PREV. GRADES, AND QJ. FOR GRADE NINE

(MALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.31	1.00					
AR	.27	.52	1.00				
AC	.41	.68	.61	1.00			
RD	.45	.42	.44	.54	1.00		
VC	.33	.27	.35	.42	.73	1.00	
IQ	.43	.45	.41	.58	.54	.47	1.00

(FEMALES)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.37	1.00					
AR	.41	.54	1.00				
AC	.48	.57	.62	1.00			
RD	.38	.48	.61	.61	1.00		
VC	.29	.40	.47	.57	.76	1.00	
IQ	.49	.52	.52	.49	.56	.48	1.00

(TOTAL)

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.31	1.00					
AR	.35	.52	1.00				
AC	.46	.60	.62	1.00			
RD	.40	.46	.53	.57	1.00		
VC	.33	.32	.43	.51	.74	1.00	
IQ	.47	.47	.47	.53	.54	.48	1.00

Code:
- QJ - Test of Quantitative Judgment
- PG - Previous Mathematics Grades
- AR - Iowa Arithmetic Reasoning
- AC - Iowa Arithmetic Concepts
- RD - Iowa Reading Comprehension
- VC - Iowa Vocabulary
- IQ - California Test of Mental Maturity (CTMM)

TABLE 34

MATRIX TO SHOW INTERCORRELATIONS BETWEEN THE CTMM, ITBS, PREV. GRADES, AND QJ. FOR TOTAL SAMPLE

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.43	1.00					
AR	.41	.64	1.00				
AC	.50	.69	.70	1.00			
RD	.47	.57	.60	.65	1.00		
VC	.46	.51	.51	.58	.78	1.00	
IQ	.42	.56	.52	.57	.59	.55	1.00

Code:

QJ - Test of Quantitative Judgment
PG - Previous Mathematics Grades
AR - Iowa Arithmetic Reasoning
AC - Iowa Arithmetic Concepts
RD - Iowa Reading Comprehension
VC - Iowa Vocabulary
IQ - California Test of Mental Maturity (CTMM)

Results of The Least-Squares and Maximum Likelihood Program. In order to test the remaining three hypotheses (2, 3, and 4) a random sample of two hundred cases from the original population was chosen. This sample consisted of twenty-five cases of each sex per grade level used in the original population. (The equal numbers for sex and grade level were chosen to meet the constraints of the program available for the analysis of the remaining data.)

The results reported in Tables 35-39 revealed the following information: (1) Children's ability to deal with aspects of quantitative judgment increased significantly with each grade level; (2) The ability to deal with aspects of quantitative judgment was found to be significantly higher in favor of the males in the population; (3) There existed no significant interaction effects between grades and sex; (4) The intelligence quotient and the score on the Iowa Arithmetic Concepts Test was significantly related to children's ability with aspects of quantitative judgment; (5) Children's ability to deal with aspects of quantitative judgment was not significantly related to their performance on the Iowa Tests of Vocabulary, Reading, and Arithmetic Reasoning; and (6) Furthermore, children's ability to deal with aspects of quantitative judgment was not significantly related to previous mathematics grades.

TABLE 35

DISTRIBUTION OF CLASS AND SUBCLASS NUMBERS FOR THE LEAST-SQUARES ANALYSIS

Identification	Number
Grade 6	50
Grade 7	50
Grade 8	50
Grade 9	50
Male	100
Female	100
Grade Six Males	25
Grade Six Females	25
Grade Seven Males	25
Grade Seven Females	25
Grade Eight Males	25
Grade Eight Females	25
Grade Nine Males	25
Grade Nine Females	25

TABLE 36

OVERALL MEANS AND STANDARD DEVIATIONS OF THE VARIABLES USED IN THE LEAST-SQUARES ANALYSIS

Variable	Mean	S.D.
IQ	110.915	13.963
Vocabulary	61.980	23.661
Reading	60.710	24.012
Arith. Concepts	65.260	23.775
Arith. Reasoning	62.410	26.328
Prev. Grades	32.950	9.393
QJ	39.400	5.905

TABLE 37

MATRIX TO SHOW INTERCORRELATIONS BETWEEN THE CTMM, ITBS, PREV. GRADES, AND QJ FOR SAMPLE IN THE LEAST-SQUARES ANALYSIS

	QJ	PG	AR	AC	RD	VC	IQ
QJ	1.00						
PG	.39	1.00					
AR	.41	.67	1.00				
AC	.49	.69	.74	1.00			
RD	.49	.62	.63	.69	1.00		
VC	.44	.50	.50	.54	.81	1.00	
IQ	.48	.57	.53	.58	.64	.61	1.00

Code:
QJ - Test of Quantitative Judgment
PG - Previous Mathematics Grades
AR - Iowa Arithmetic Reasoning
AC - Iowa Arithmetic Concepts
RD - Iowa Reading Comprehension
VC - Iowa Vocabulary
IQ - California Test of Mental Maturity (CTMM)
ITBS- Iowa Tests of Basic Skills

TABLE 38

LISTING OF CONSTANTS, LEAST-SQUARES MEANS, AND STANDARD ERRORS FOR THE LEAST-SQUARES ANALYSIS

Variables	No. Obs.	Constant Estimate	Least-Squares Mean	Standard Error
QJ Mean	200	39.400	39.400	0.3136
Grade 6	50	-2.546	36.853	0.6439
Grade 7	50	-1.008	38.391	0.6452
Grade 8	50	0.934	40.334	0.6392
Grade 9	50	2.620	42.020	0.6433
Sex (Male)	100	1.192	40.592	0.4521
Sex (Female)	100	-1.192	38.207	0.4521
Grades x Sex 6M	25	0.752	38.799	0.9134
Grades x Sex 6F	25	-0.752	34.908	0.9026
Grades x Sex 7M	25	-0.721	38.862	0.9021
Grades x Sex 7F	25	0.721	37.920	0.9002
Grades x Sex 8M	25	0.489	42.017	0.9165
Grades x Sex 8F	25	-0.489	38.652	0.9068
Grades x Sex 9M	25	-0.520	42.691	0.9126
Grades x Sex 9F	25	0.520	41.348	0.9113
Lin. Reg. IQ		0.117		0.0327
Lin. Reg. VC		0.014		0.0236
Lin. Reg RD		0.027		0.0275
Lin. Reg. AC		0.053		0.0237
Lin. Reg. AR		0.004		0.0199
Lin. Reg. PG		0.033		0.0522

Code: Lin. Reg. - Linear Regression
QJ - Test of Quantitative Judgment
IQ - California Test of Mental Maturity (CTMM)
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithemtic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades
M - Male
F - Female

TABLE 39

LEAST-SQUARES ANALYSIS OF VARIANCE

Source	df	Sum of Squares	Mean Squares	F
Total (Deviation)	200	6942.000		
Total Reduction	14	3282.765	234.483	11.919**
Mu - $\overline{X}$m	1	2.000	2.000	0.102
Grades	3	725.601	241.867	12.294**
Sex	1	263.813	263.813	13.410**
Grades x Sex	3	77.047	25.682	1.305
IQ BLinear	1	253.813	253.813	12.870**
VC BLinear	1	7.082	7.082	0.360
RD BLinear	1	20.309	20.309	1.032
AC BLinear	1	98.674	98.674	5.016**
AR BLinear	1	1.025	1.025	0.052
PG BLinear	1	8.060	8.060	0.410
Remainder (Error)	186	3659.234	19.673	

Code:

IQ - California Test of Mental Maturity (CTMM)
VC - Iowa Vocabulary
RD - Iowa Reading Comprehension
AC - Iowa Arithmetic Concepts
AR - Iowa Arithmetic Reasoning
PG - Previous Mathematics Grades

** significant at the .01 level

CHAPTER V

SUMMARY AND CONCLUSIONS

The purpose of this study was to investigate the ability of upper elementary and junior high school students on aspects of quantitative judgment. The following basic questions were implied or asked in the study: (1) Is the Test of Quantitative Judgment measuring a unique attribute? (2) Is there a significant difference in the performance between sexes when dealing with aspects of quantitative judgment? (3) Does an increase in grade level (or age) influence the performance on aspects of quantitative judgment? (4) Is there any significant interaction effects between grades and sex? (5) What significant relationships exist between the scores on the Test of Quantitative Judgment and the other variables used in the study?

Findings of the Study

The writer found that the Test of Quantitative Judgment was measuring a unique attribute which was significantly related to one's intelligence quotient and score on the Iowa Test of Arithmetic Concepts. There was a statistically significant difference in performance between sexes when dealing with quantitative judgment as sampled in the study. The males scored significantly higher than the females. A statistically significant difference in ability between grades was also found. The study revealed that the scores on the aspects of quantitative judgment used in the study increased as the grade level increased. Furthermore, the grades x sex interaction effects for the Test of Quantitative Judgment were not significant (ie., there existed no pair (grade, sex) whose score was statistically significant from the other pairs).

The correlations between quantitative judgment and the other variables used in the study (reported in Tables 30-34) may be summarized as follows: (1) The correlation between the intelligence test scores and the scores on the Test of Quantitative Judgment was higher for girls than for boys with slight decreases in the correlation coefficient as the grade level increased; (2) The correlation coefficients of previous grades with the scores on the Test of Quantitative Judgment decreased as the grade level increased; (3) The correlation coefficients between the scores on the Test of Quantitative Judgment and the scores on the ITBS (reading, vocabulary, and arithmetic reasoning also decreased as the grade level increased; and (4) The coefficient of correlation between the scores on the ITBS Arithmetic Concepts Test and the scores on the Test of Quantitative Judgment remained high and fairly constant until decreases occurred at the ninth grade level.

Limitations of the Study

The findings and conclusions from this study would be applicable only for the aspects of quantitative judgment contained in the Test of Quantitative Judgment, Form T, which may be found in the appendix.

The sample used included an equal mixture of average and above-average ability groups as defined by the school system. Students from many sections of the community were represented in the study. The results of this study pertain only to the members of this sample or at most to the levels of the school community represented in this study.

There were over 200 cases at each grade level in the study. The entire study consisted of a population of 884 cases.

Conclusions

Within the limits of this investigation the following conclusions are made: (1) There exists an important amount of variance remaining after removing the variation accounted for by the following:

Q.J. - (Variance of the California Test of Mental Maturity)
Q.J. - (Variance of the parts of the Iowa Test of Basic Skills)(Vocabulary, Reading, Arithmetic Concepts, and Arithmetic Reasoning)
Q.J. - (Variance contributed by previous mathematics achievement grades for the previous two years);

(2) Children's ability to deal with the aspects of quantitative

judgment included in this study will differ or will vary directly with grade level (or age); (3) Children's ability to deal with the aspects of quantitative judgment included in this study will differ over the variable sex; (4) There is a significant relationship between children's ability to deal with the aspects of quantitative judgment included in this study and their intelligence quotient as measured by the CTMM; (5) There is a significant relationship between children's ability to deal with the aspects of quantitative judgment included in this study and their score on the ITBS Arithmetic Concepts Test; (6) The grades x sex interaction effects were not statistically significant; (7) The estimated reliabilities seem adequate but one must note the rather large decrease in the coefficients at the ninth grade level; (8) Most of the questions contained in the Test of Quantitative Judgment seem to discriminate in the appropriate manner between high and low scorers; and (9) The gain scores (increases in means) between grades on the Test of Quantitative Judgment suggest that some aspects of quantitative judgment may be learned.

Suggestions for Further Research

(1) Improve the instrument for measuring quantitative judgment by revising and adding other suitable test items as a result of the item analysis.

(2) Perform a factor analysis on the Test of Quantitative Judgment.

(3) Conduct further pilot studies comparing the Test of Quantitative Judgment with the factors measured by other standardized tests such as the Differential Aptitude Test and the Academic Promise Test.

(4) Conduct a study with the Test of Quantitative Judgment on a sample where the students are studying a mathematics curriculum such as the one devised by The School Mathematics Study Group.

(5) Conduct a study to find the relation between the scores on the Test of Quantitative Judgment and the socioeconomic status of the members of the sample.

(6) An attempt should be made to equalize the number of questions contained in the Test of Quantitative Judgment measuring the same aspect aspect of quantitative judgment.

APPENDIX

TEST OF QUANTITATIVE JUDGMENT Form T

By: Donald E. Hall

Directions:

Read the question and the answers that are below it. Choose the answer you think is correct and darken the correct choice on the answer sheet. Consider the sample questions and see how they are done.

SAMPLE QUESTIONS

1. Twenty-five cents is about enough to buy:
   1. a fur coat
   2. a quart of milk
   3. a new silk dress
   4. two pounds of steak

2. To hard-cook an egg, boil it for:
   1. 10-12 minutes
   2. 3 minutes
   3. 2-3 minutes
   4. 1 minute

SAMPLE ANSWER SHEET

IBM Answer Card

1	2	3	4
(1)	(1)	(1)	(1)
(2)	(2)	(2)	(2)
(3)	(3)	(3)	(3)
(4)	(4)	(4)	(4)
(5)	(5)	(5)	(5)
(T)	(T)	(T)	(T)
(F)	(F)	(F)	(F)

Use only choices 1, 2, 3, or 4.
<u>DO NOT USE choices 5, T, or F.</u>

1. Ten-year-old Jim took 1 giant step, 1 hop, and 1 jump. He probably covered a distance of about
   1. 10 feet
   2. 20 feet
   3. 30 feet
   4. 40 feet

2. The desk was 3 feet long. Its width was about:
   1. 1 foot
   2. 2 feet
   3. 3 feet
   4. 4 feet

3. Cindy pushed the window open a crack. To her surprise a guest came in. It probably was:
   1. an elephant
   2. a sparrow
   3. an airplane
   4. a bee

4. Sally did some shopping. She bought a dozen eggs, two quarts of milk, a large box of soap chips, two packages of cereal, and two jars of jelly. The bill probably amounted to:
   1. $3.50
   2. $1.00
   3. $15.00
   4. $ .78

5. Mr. Green stood on the deck of his boat and saw another boat about 1/2 mile away. He probably could also see which one of the following things?
   1. A sailor lighting a cigarette
   2. a flag flying in the breeze
   3. a girl reading a book
   4. a kitten playing on deck

6. An apple sliced in halves measured 3 inches on one of the flat surfaces. To hold the apple together again you would need one piece of tape just over:
   1. 16 inches
   2. 9 inches
   3. 3 inches
   4. 1 inch

7. Jill's mother told her to wear her winter coat because the thermometer read:
   1. 100 degrees
   2. 70 degrees
   3. 50 degrees
   4. 20 degrees

8. A candy bar weighs about:
   1. 3 grams
   2. 3 pounds
   3. 3 ounces
   4. 3 cubits

9. One would have the greatest number of things in a pail of:
   1. apples
   2. eggs
   3. potatoes
   4. grapes

10. Box A is 2 feet high, 2 feet wide, and 6 feet long. Box B is 6 feet high, 2 feet wide, and 2 feet long. If Box A is filled with sand and then poured into Box B, the latter is:
    1. exactly full
    2. overflowing
    3. half-full
    4. two-thirds full

11. A person with normal eyesight usually holds a book about how far away from him while reading?
    1. 12-14 inches   3. 3 inches
    2. 36-38 inches   4. 1 inch

12. A boy went fishing and caught 4 fish. The first one measured 6 inches; the second, 8 inches; the third, 10 inches; the fourth probably measured:
    1. 15 inches   3. 3 inches
    2. 12 inches   4. 11 inches

13. Last summer Jim was 49 inches tall; at Christmas he was 52 inches tall. Next summer he will probably be how tall?
    1. 52 inches   3. 55 inches
    2. 53 inches   4. 59 inches

14. The cookie dish was about 3 1/2 feet around the outside. It was about how far across?
    1. 1 foot   3. 3 feet
    2. 2 feet   4. 4 feet

15. Mr. Jones found the bunk-bed in his cabin too small for him to he had to sleep on the floor. The length of the bed was probably:
    1. 5 feet   3. 1 foot
    2. 10 feet   4. 7 feet

16. Uncle Ben takes his lunch to work with him. The average sized thermos bottle for his coffee holds about:
    1. 1 gallon   3. 1/2 gallon
    2. 3 quarts   4. 1 pint

17. A pound of tomatoes might fit into the same sized bag as:
    1. a pound of nails   3. a pound of feathers
    2. a pound of apples   4. none of these

18. Lou punched holes close to one another around the outside edge of a post card. She found that she could punch about:
    1. 1-5 holes   3. 35 holes
    2. 15 holes   4. 100 holes

19. A quart jar filled with which of the following would be heaviest to carry:
    1. sand   3. water
    2. leaves   4. milk

20. In first grade Mary weighed 56 pounds and Sue weighed 40 pounds. Mary looked fatter than Sue because:
    1. she wore a blue dress   3. she was two inches shorter
    2. she was 2 inches taller   4. she had black hair

21. A new lead pencil is about:
    1. 3 inches long 3. 4-5 inches long
    2. 7-8 inches long 4. 1 foot long

22. Stan decided to sell eggs for 70 cents a pound instead of 70 cents a dozen. He probably made:
    1. the same amount of money 3. More money
    2. less money 4. none of the above

23. Jim's father was stopped for speeding on the turnpike. How many miles per hour was his car probably going?
    1. 25 3. 75
    2. 50 4. 125

24. A 12-year-old boy can just about lift a stone, but it is a little too heavy for him to carry. It is probably the size of:
    1. a baseball 3. a bale of hay
    2. a basketball 4. a small pony

25. A half-inch is about the thickness of:
    1. a slice of bread 3. a potato chip
    2. a slice of Swiss cheese 4. a sheet of paper

26. A tailor made a suit for a man. About how much wool did he use?
    1. 1 yard 3. 6 yards
    2. 3 yards 4. 12 yards

27. John can walk to the Junior High School in 10 minutes. John walks a distance of:
    1. 1 mile 3. 3 miles
    2. 1/2 mile 4. 10 miles

28. Jim's Dad built him a glass case for his ship model. It was 12 long, 4 wide, and 6 high. These are probably:
    1. feet 3. yards
    2. rods 4. inches

29. There are as many stars in the sky as there are grains of sand on all of the beaches in the whole world. In the sky then, there are probably about:
    1. 100-200 stars 3. 20 billion stars
    2. 5000-8000 stars 4. no one knows

30. Don and his father drove directly from Boston to Florida in:
    1. two minutes 3. two days
    2. two hours 4. two weeks

31. Rick stared out his hotel window and saw the cars below which looked like toys. He was staying on the:
    1. second floor
    2. fourth floor
    3. eighth floor
    4. sixteenth floor

32. Peggy poured some milk over her cereal. She probably used how much milk:
    1. one cup
    2. one pint
    3. one-half cup
    4. two teaspoonfuls

33. A gallon of gasoline is enough fuel for an American car to travel:
    1. 2640 feet
    2. 80 yards
    3. one mile
    4. 15 miles

34. A boy in the sixth grade read a 55-page book. It probably took him:
    1. 30 minutes
    2. 2 hours
    3. 3 days
    4. 55 seconds

35. Jim built a house which measured 6' by 4' by 4' in length, width, and height for his pet. His pet was probably:
    1. a bird
    2. a cat
    3. a horse
    4. a dog

36. Mother asked Jim to ride his bicycle to the store for a loaf of bread. The store was two miles away, so he probably got back home in about:
    1. 1/2 hour
    2. 1 1/2 hours
    3. 4 hours
    4. 7 1/2 hours

37. Harry received $50. 00 from Uncle Jim as a birthday present. Uncle Jim asked him to buy just one thing with the money. What did Harry buy?
    1. a bicycle
    2. two new books
    3. fur mittens
    4. a motor boat

38. Tom had a map of a campsite on which 6 inches equaled one mile. He drew another map twice as large. Then 6 inches equaled:
    1. 4 miles
    2. 2 miles
    3. 1/2 mile
    4. 1/4 mile

39. Sally-a-fifth-grader made a jump rope. For this she needed a piece of rope about how long?
    1. 1 rod
    2. 2 yards
    3. 3 feet
    4. 12 inches

40. It was steady cold for 15 days in January, and every 5 days Tom kept measuring the ice on the pond. The first time it measured 3 inches, next 5 inches, then 7 inches. In five more days it probably measured:
    1. 15 inches
    2. 10 inches
    3. 12 inches
    4. 9 inches

41. A Southern farmer said: "It takes <u>much time</u> for cotton plants to grow." By <u>much time</u> he meant:
1. an hour
2. 3 months
3. 5 years
4. 12 1/2 minutes

42. There are three trees. If the third tree is twice as high as the second, and the second tree is twice as high as the first, the third tree is ______ as high as the first.
1. three times
2. four times
3. six times
4. eight times

43. Which of the following would take up the most room on a shelf?
1. a box of dried prunes
2. a tube of toothpaste
3. a pound of mercury
4. a gallon of milk

44. A mile is about:
1. the thickness of a book
2. 1760 feet
3. the distance a man can walk in fifteen minutes
4. as long as a ball field

45. In a pound of peanuts and a pound of bananas there would be:
1. more bananas than peanuts
2. more peanuts than bananas
3. about the same number of each
4. none of these

46. Sam read a 300 page book. It was about as thick as:
1. a potato chip
2. a pancake
3. a brick
4. a cheese sandwich

47. John was sick from eating too much ice cream. He probably ate:
1. 2 cones
2. 6 spoonfuls
3. 3 dishfuls
4. 7 quarts

48. Joe's kite got tangled in wires on top of the telephone pole. How tall a ladder will be needed to get the kite?
1. 20 feet
2. 100 feet
3. 7 feet
4. 47 feet

49. John read in a book that the last dinosaur died about 65,000,000 years ago. This was about the same time as:
1. the birth of Abraham Lincoln
2. before the cave man
3. 1000 after the cave man
4. when Washington was president

50. Fred, a fifth grader, works after school lifting heavy boxes onto a truck. They probably weigh:
1. 50 pounds
2. 5 pounds
3. 100 pounds
4. 25 pounds

51. The elevator held fifteen people. The floor measured about:
    1. 1 foot by 3 feet
    2. 2 feet by 4 feet
    3. 5 feet by 8 feet
    4. 16 feet by 20 feet

52. A bird flew non-stop from one city to another 10 miles away. It probably took about:
    1. 1 week
    2. 1 day
    3. 1 hour
    4. 5 minutes

53. A man bought a new bicycle for his son. It probably cost:
    1. $5.95
    2. $195.95
    3. $11.50
    4. $59.95

54. John used to row across the pond in one hour. Half of the water was let out; it now takes him:
    1. 45 minutes
    2. 30 minutes
    3. 15 minutes
    4. 2 hours

55. Fran filled one quart jar with grapes and the other with apples. He had:
    1. more apples that grapes
    2. more grapes than apples
    3. the same amount of each
    4. none of the above

56. A boy picked a hat full of peas. He probably had:
    1. a quart
    2. a pint
    3. 1/2 peck
    4. a bushel

57. Jim's Dad built him a new boat. It was 10 feet long. About how wide is it?
    1. 1 foot
    2. 12 feet
    3. 3 feet
    4. 7 inches

58. Tom can mow his lawn in 20 minutes. His grandmother's lawn is twice as wide and twice as long. He can probably mow her lawn in:
    1. 20 minutes
    2. 40 minutes
    3. 60 minutes
    4. 80 minutes

59. Nancy started cracking eggs into a measuring cup. She found that it was full after she had cracked about:
    1. 1 egg
    2. 2 eggs
    3. 4 eggs
    4. 8 eggs

60. Tomatoes come from the market in boxes that are 3 inches wide and 12 inches long. They hold just 4 <u>large</u> tomatoes. If the boxes were <u>twice as wide and twice as long</u> they would hold:
    1. 8 tomatoes
    2. 12 tomatoes
    3. 16 tomatoes
    4. 24 tomatoes

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